Chapter 6

Not all transitions are equal:

The relationship between inequality of educational opportunities and inequality of educational outcomes

6.1 Introduction

Social stratification research has long been concerned with the relationship between family socioeconomic status (SES) and offspring's educational attainment (Breen and Jonsson, 2005; Hout and DiPrete, 2006). A strong positive association between the two implies that respondents with higher SES backgrounds are more likely to achieve higher levels of education than respondents with lower SES backgrounds. For this reason, the strength of the relationship is often termed 'Inequality of Educational Opportunity' or IEO (Boudon, 1974; Mare, 1981). IEO can be measured in a variety of ways, and these different measures tend to lead to seemingly different conclusions. This chapter will focus on two of these measures of IEO: the association between family SES and the highest achieved level of education, and the association between family SES and probabilities of passing from one educational programme to the next. These will be called Inequality of Educational Outcome (IEOut) and Inequality of Educational Opportunity proper (IEOpp) respectively, while IEO will be used as a generic term. IEOut focusses on the end result of the educational process, which is often of interest as this result, the highest achieved level of education, is the most visible result of education in subsequent areas of life like the labor market or the marriage market. IEOpp focusses on the process of attaining education. Attaining a level of education is something that typically happens over a long period of time and is usually split up into different steps, for example finishing primary eduction, finishing secondary education, etc. Knowing the influence of SES at each of these transitions can give a more complete picture of how IEO came about. So, these two measures of IEO capture different aspects of IEO: IEOut describes inequality of the outcome of the process of attaining education, while IEOpp describes inequalities in that process itself. The aim of this chapter is to show how estimates of IEOpp and IEOut can complement one another. The key challenge when dealing with complementary models is to find a way to move beyond just presenting separate results from different models to an integrated discussion of the results that shows how the different results are related to one another.

This is done by demonstrating that there is a relationship between IEOpp and IEOut in the form of a decomposition of IEOut as a weighted sum IEOpps. This means that the IEOpps (the process) lead to IEOut (the outcome), but that not every IEOpp (that is, every step in the process) is of equal importance for achieving the outcome. Moreover, as will be shown below, the importance of each IEOpp for the IEOut can differ across groups. A clear example of this is the differences in the importance of the transition between primary education and secondary education between cohorts. In most industrialized countries virtually all students within the recent cohorts remain in education after the primary level. As a result, any inequality at this first transition only affects a few (or no) students, and is thus not very important for IEOut. The situation was quite different at the beginning of the twentieth century: at that time many more students failed to continue after primary education, so the IEOpp for the transition between primary and secondary education was much more important for the IEOut than it is now. Within the decomposition developed in this chapter there will be two additional reasons why the importance of a transition can differ across groups: the importance of a transition will increase as the proportion of people at risk increases, and when the difference in the value of the expected highest attained level of education between those that pass and those that fail increases. All three are substantively interpretable ways in which the distribution of education — that is, for each educational programme the proportion of people that has that program as their highest achieved level of eduction — can influence IEOut. This decomposition thus leads one to relate IEOpp and IEOut to one another as two complementary descriptions of IEO, and allows one to investigate the effect of changes in the distribution of education on IEOut. The fact that IEOut and IEOpp are related is not new, Mare (1981) already established that, but the use of this relationship to create an integrated analysis of IEOpp and IEOut and to study the impact of educational expansion on IEOut is new to the best of my knowledge.

This chapter will begin with a description of a number of models of educational inequality. This will be followed by a discussion of the model proposed by Mare (1981), and the derivation of the relationship between IEOpp, IEOut, and the distribution of education. In the next section the decomposition will be illustrated by applying it to differences in IEOut between men and women and across cohorts that were 12 years old in the Netherlands between 1905 and 1991.

6.2 Different models of IEO

A variety of different models have been proposed and used for studying IEO. These different models tend to emphasize different aspects of IEO. For example much of the early research focusses on inequality in the end result by studying the association between family background and highest achieved level of education (Blau and Duncan, 1967; Duncan, 1967; Hauser and Featherman, 1976). This research was supplemented by Boudon (1974) and Mare (1980, 1981), who studied educational inequality during the process of attaining education as the effect of family background on the probability of passing steps between educational programmes. In particular, Mare (1980, 1981) proposed the use of the sequential logit model for estimating IEOpps. Estimates of IEOpp and IEOut are now often treated as competing representations of educational inequality. The reason for that is that Mare (1981) showed that there is an relationship between IEOpp and IEOut which involved the transition probabilities, but presented this relationship as a black box. The main point he made was that differences in these estimates of IEOut between cohorts are in part due to differences in the distribution of education. These effects can be considerable, since the distribution of education varies substantially over cohorts. In almost all countries, people born in later cohorts have attained more education, a process that has been termed ‘educational expansion’ (Hout and DiPrete, 2006). Furthermore, Mare (1981) showed the IEOpps control for this effect of educational expansion. This led Mare (1981) to argue that the IEOpps are a more ‘pure’ measure of IEO. Since then, the literature has approached the relationship between IEOut, the IEOpps, and the distribution of education as a black box.

This practice leads one to ignore two opportunities. First, the complementary nature of the information contained in estimates of IEOpps and IEOut are not fully used when treating the relation between these two as a black box. IEOpp and IEOut are natural complements as the former describes the process of attaining education while the latter describes the outcome of that process. Some studies report both estimates for the IEOpps and the IEOut, (for example Shavit and Blossfeld, 1993) but these do not relate the two types of estimates to one another. Second, this practice makes it hard to study the impact of educational expansion on IEO, because one explicitly controls for changes in the distribution of education. Those studies that have investigated the relationship (Mare, 1981; Smith and Cheung, 1986; Nieuwbeerta and Rijken, 1996) compare the observed IEOut with the simulated results of two counterfactual scenarios, those being that either the distribution of education remained unchanged and IEOpp changes as observed; or that the distribution of education changes as observed, but IEOpp remains unchanged. Simulations such as these can tell us how much IEOut is affected by changes in the distribution of education and changes in IEOpps, but do not offer us any insights as to why. This leads to the following two questions:

> How are IEOut and IEOpp related to one another, and how can this relation be used for a meaningfully integrated analysis of IEOpp and IEOut?
>
> How are IEOut and the distribution of education related to one another, and how can this relation be used for an analysis of the influence of changes in the distribution of education on IEOut?

These questions are answered by showing that the standard model for estimating IEOpps, the sequential logit model proposed by Mare (1981), implies an estimate of IEOut, which can be decomposed into a weighted sum of the IEOpps. Moreover, it will be shown that each IEOpp's weight depends on the distribution of education in three substantively interesting ways. An IEOpp receives more weight if 1) the proportion of people 'at risk' of making that transition increases; 2) the proportion passing that transition is closer to 50%, that is, passing or failing that transition cannot be regarded as almost universal; and 3) the difference in expected level of education between those who pass and those who fail to make the transition increases, that is, the expected gain from passing increases. This decomposition of IEOut into a weighted sum of IEOpps provides a link between IEOpp and IEOut and a way of conducting an integrated analysis of the two. The decomposition of the weights into the product of its three elements provides a link between the distribution of education and IEOut and a way of showing the influence of changes in the distribution of education on IEOut. The decomposition of IEOut into IEOpps and weights has been implemented in Stata (StataCorp, 2007) in the `seqlogit` package (Buis, 2007b), which is documented in Technical Materials II.

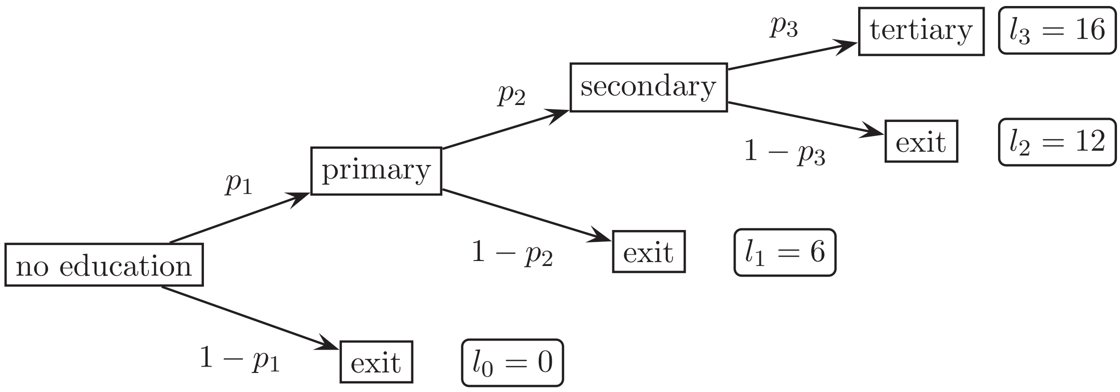
This decomposition does not require a new model, it is just a different way of presenting the results of a sequential logit model. This means that the critique by Cameron and Heckman (1998) on the sequential logit model also applies to this decomposition. Their argument starts with the observation that it is very likely that not all variables that influence the probability of passing a transition are observed. In this case the sequential logit model will estimate the effect of the observed explanatory variables on the proportion of respondents that pass a transition averaged over these unobserved variables rather than on an individual's probability of passing the transitions. The problem is that the group level effects measured by the sequential logit model will not be the same as the individual level effects, even if the unobserved variables are non-confounding variables. The easiest solution is to interpret the results of the sequential logit model as a description of differences between different groups rather than interpret the results as individual-level effects. Alternatively, one can try to adapt the model to take unobserved heterogeneity into account. This is obviously a difficult problem, as one tries to control for variables that have not been observed, and a consensus on the best way of doing this has yet to appear. A discussion of the various solutions proposed to solve this problem is beyond the scope of this chapter, so the main focus of this chapter will be on the effects on group-level transition rates rather than individual-level effects. However, the decomposition can be applied to some of the models that have been proposed for estimating individual-level effects (for example: Mare 1993 and Chapter 7 of this dissertation), and generalizations of the decomposition for these models will be briefly discussed.

6.3 The relationship between inequality of educational opportunities and outcomes

In this section I will derive and discuss a decomposition of an estimate of IEOut into a weighted sum of IEOpps. This decomposition starts with the model for IEOpps proposed by Mare (1981), which I will refer to as the sequential logit model (following Tutz (1991)). This model is also known under a variety of other names: sequential response model (Maddala, 1983), continuation ratio logit (Agresti, 2002), model for nested dichotomies (Fox, 1997), and simply the Mare model (Shavit and Blossfeld, 1993). Consider, for instance, a hypothetical education system consisting of four levels: no education, primary education, secondary education, and tertiary education as represented in Figure 6.1. Figure 6.1 shows how respondents face three transitions in this system: they can attend primary education or opt for no education at all; if they opt for primary education they can choose to leave the system once they have completed primary education, or go on to secondary education; and if they opt for secondary education, they can then either choose to leave once they have completed this level or go on to tertiary education. The implication is that if someone's highest-achieved level of education is primary education, then that person was 'at risk' of passing the first two transitions, but not the third. Furthermore, it implies that the person passed the first transition, but failed the second.

The model assumes that one has to be 'at risk' of passing a transition — that is, to have passed through all lower transitions — in order to make a decision at that transition about whether to continue in education or to leave the system. Aside from this, these decisions are assumed to be completely independent. As a result, one can estimate the IEOpp by running separate logistic regressions for each transition on the appropriate sub-sample (Mare, 1980). This model is shown in equation (6.1).

Figure 6.1: Hypothetical educational system



$$
\begin{aligned}
\hat{p}_{1i} &= \frac{\exp(\alpha_1 + \lambda_1 SES_i + \beta_1 x_i)}{1+\exp(\alpha_1 + \lambda_1 SES_i + \beta_1 x_i)} \\
\hat{p}_{2i} &= \frac{\exp(\alpha_2 + \lambda_2 SES_i + \beta_2 x_i)}{1+\exp(\alpha_2 + \lambda_2 SES_i + \beta_2 x_i)} \quad \text{if} \quad pass_{1\,i} = 1 \\
\hat{p}_{3i} &= \frac{\exp(\alpha_3 + \lambda_3 SES_i + \beta_3 x_i)}{1+\exp(\alpha_3 + \lambda_3 SES_i + \beta_3 x_i)} \quad \text{if} \quad pass_{2\,i} = 1
\end{aligned}
\tag{6.1}
$$

The probability that person i passes transition k is $\hat{p}_{ki}$. The IEOpp belonging to transition k is λ_k, the constant for transition k is α_k, and the effect of a control variable x_i is represented by β_k. Whether or not individual i has passed the previous transition is indicated by the indicator variable $pass_{k-1\,i}$. It is assumed that everybody is at risk of passing the first transition. The differences in IEOpp between men, women, and cohorts can be obtained by adding the appropriate interaction terms to the model.

In order to make a link between the IEOpps (the λ_ks) and IEOut, it is necessary to assign a value (l_k) to each level of education. By assigning values to each educational level, it becomes possible to use the sequential logit model to calculate the expected highest achieved level of education ($E(L_i)$). The results from the sequential logit are used to compute predicted probabilities for passing each transition, and the expected highest achieved level of education is the sum of the value of each level of education times the probability of attaining that level. This is set out in equation (6.2). The probabilities and values assigned to each level can be derived from Figure 6.1[1].

[1]The values that are assigned to each of the levels in Figure 6.1 are typical for when these values are based on years or pseudo-years of education, but this decomposition is not limited to this metric.

$$E(L_i) = (1 - \hat{p}_{1i})l_0 + \hat{p}_{1i}(1 - \hat{p}_{2i})l_1 + \hat{p}_{1i}\hat{p}_{2i}(1 - \hat{p}_{3i})l_2 + \hat{p}_{1i}\hat{p}_{2i}\hat{p}_{3i}l_3 \quad (6.2)$$

The family's SES is part of equation (6.2) through the $\hat{p}_{ki}$s described in equation (6.1). Equation (6.2) can be understood as a regression equation showing a non-linear relationship between a family's SES and the highest achieved level of education. Using a sequential logit model to derive such a (non-linear) regression is unusual. A more common method for estimating IEOut is to use a linear regression of highest achieved level of education on family SES (for example, Blau and Duncan, 1967; Shavit and Blossfeld, 1993). The advantage of the non-linear model derived from the sequential logit model over the linear model is that the non-linear model provides the link between the IEOpps and the IEOut. Moreover, the non-linear model takes the bounded nature of the dependent variable into account, as it can never lead to predictions below the lowest level of education or above the highest level of education.

Recall that IEOut is the effect of a family's SES on the respondent's expected highest achieved level of education, or, in other words, how much the expected highest achieved level of education changes if a family's SES changes[2]. Consequently, IEOut is the first derivative of equation (6.2) with respect to a family's SES. This derivative is shown in equation (6.3). A step-by-step derivation is set out in the appendix to this chapter.

$$\begin{array}{lllllll} \frac{\partial E(L_i)}{\partial SES} & = & & & & & \\ \{1 & \times & \hat{p}_{1i}(1-\hat{p}_{1i}) & \times & [(1-\hat{p}_{2i})l_1 + \hat{p}_{2i}(1-\hat{p}_{3i})l_2 + & & \\ & & & & \hat{p}_{2i}\hat{p}_{3i}l_3 - l_0] & \} & \lambda_1 \quad + \\ \{\hat{p}_{1i} & \times & \hat{p}_{2i}(1-\hat{p}_{2i}) & \times & [(1-\hat{p}_{3i})l_2 + \hat{p}_{3i}l_3 - l_1] & \} & \lambda_2 \quad + \\ \{\hat{p}_{1i}\hat{p}_{2i} & \times & \hat{p}_{3i}(1-\hat{p}_{3i}) & \times & [(l_3 - l_2)] & \} & \lambda_3 \end{array} \quad (6.3)$$

Equation (6.3) shows that IEOut $\left(\frac{\partial E(L_i)}{\partial SES}\right)$ is a weighted sum of the IEOpps (the λ_ks). The weights (the sections between curly brackets) consist of three parts, all of which are related to the distribution of education. These are:

1. The predicted proportion of people at risk of passing a transition. For the first transition, this proportion is 1; for the second it is the proportion of students who complete primary education, $\hat{p}_{1i}$; and for the third transition, it is the proportion who completed secondary education, $\hat{p}_{1i}\hat{p}_{2i}$. Substantively, this means that a transition is more important when more people are at risk of passing it.

[2]More precisely, the measure of IEOut used in this chapter studies how the average highest achieved level of education of a group of respondents with the same family SES reacts to a change in the family SES rather than an individual-level effect, as was discussed before.

2. The variance of the indicator variable showing who passed and who failed the transition, $\hat{p}_{ki}(1-\hat{p}_{ki})$. This variance is a function of the predicted probability of passing. This is lowest if virtually everybody passes or fails, and is highest when the probability of passing is .5. This makes sense at a substantive level, because if only a few people pass or fail a transition, then any inequality at this stage will only affect a few people.

3. The differences between the expected level of education of those who pass the transitions and those who do not. These are the parts in the square brackets. For instance, the expected level of education of those who pass the first transition is $(1-\hat{p}_{2i})l_1 + \hat{p}_{2i}(1-\hat{p}_{3i})l_2 + \hat{p}_{2i}\hat{p}_{3i}l_3$ and the expected level of education for those that fail the first transition is l_0. The difference between the two is the expected gain from passing the first transition. The substantive interpretation of this is that a transition becomes more important if passing it leads to a greater expected increase in the highest achieved level of education.

The result is summarized below. IEOut is a weighted sum of IEOpps, and the weights are the product of the proportion at risk, the variance, and the expected gain in level of education resulting from passing.

$$\begin{aligned} \text{IEOut}_i &= \sum_{k=1}^{K} (\text{weight}_{ki} \times \text{IEOpp}_k) \\ \text{weight}_{ki} &= \text{at risk}_{ki} \times \ \text{variance}_{ki} \times \ \text{gain}_{ki} \end{aligned}$$

Each respondent will have its own IEOut and set of weights because the weights are based on the predicted probabilities of passing the transitions, and these probabilities will differ between persons depending on their values on the explanatory variables. In this chapter this decomposition will be summarized by computing the decomposition for an individual with average values on the explanatory variables. This is not the only way one can summarize the IEOuts. For example, one can compute the IEOut for each individual and average those. This 'averaged IEOut' can also be decomposed into a weighted sum of IEOpps, where the weights are now the average of the weights predicted for each individual. However, these averaged weights can no longer be decomposed as the product of its three constitutive elements[3]. This is why the IEOut of a person with average values on its explanatory variables is preferred over the 'averaged IEOut'.

[3] The reason for this is that the weight is a product of variables, and the average of a product of variables is not the same as the product of the averages of these variables.

As was discussed before, this decomposition is just a different way of representing the results from a sequential logit model, so the criticism by Cameron and Heckman (1998) also applies here. However, this decomposition can be extended to models that estimate individual-level IEOpps as long as the individual-level IEOpps are estimated by modelling the transition probabilities using a logistic curve, as is the case in (Mare, 1993) and Chapter 7. In both articles, certain assumptions are made concerning the distribution of the unobserved variables, and the IEOpps are estimated given these assumptions. The presence of the unobserved variables complicates the estimation in ways that are beyond the scope of this chapter, but within the context of equations (6.1), (6.2), and (6.3) the unobserved variable is not different from the observed variables. In this case one can create predicted probabilities for someone with average values on both the observed and unobserved variables and use those to compute the decomposition in equation (6.3).

In summary, the main advantage of the decomposition proposed in this chapter is that it allows for an integrated discussion of IEOpps and IEOut and a way of studying the influence of changes in the distribution of education on IEOut. This makes it possible to make full use of the complementary nature of IEOpp and IEOut, and to study the influence of factors such as educational expansion on IEOut. One can easily extend this argument, allowing us to study the roles played by gender educational inequality, racial educational inequality, or differences in the distribution of education between countries. A graphical representation of this decomposition is presented during the empirical discussion.

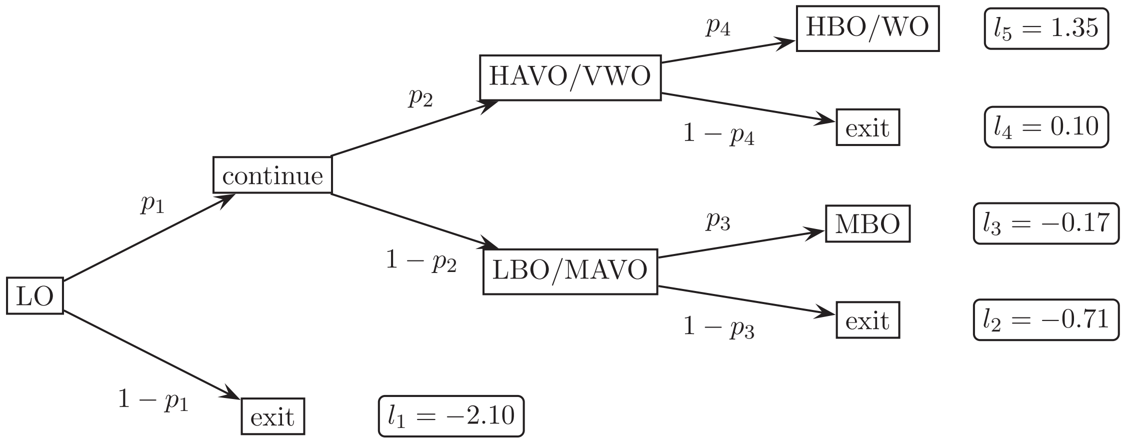
6.4 Empirical application

This section will illustrate how the relationship between IEOpp, IEOut, and the distribution of education can be used to gain a more complete picture of IEO. In particular, this section will describe the relationship between IEOpp and IEOut and the influence of educational expansion and gender inequality on IEOut in the Netherlands for cohorts that were 12 years old between 1905 and 1991.

6.4.1 The Dutch education system

The aim is to estimate a sequential logit model for the Netherlands and use the results to compute the decomposition of IEOut into IEOpps and their weights. The challenge is to come up with a model for the Dutch education system that provides a good representation of the education system during the entire period under study and where the assumption that each level can be achieved via only one route through the education system is plausible. The strategy used for meeting these challenges is to create a

Figure 6.2: Simplified model of the Dutch education system



stylized model of the Dutch education system by combining educational programmes into 'rougher' categories. This helps with keeping the model representative for the entire period, because even though the position of individual educational programmes within the Dutch education system could have changed over time, the positions of the rougher categorizations have remained reasonably stable. Using rougher categories also helps relax the assumption that each level can only be achieved through one route through the education system, as individuals are now allowed to 'move freely' within the rough categories. The stylized system is presented in Figure 6.2. The simplified representation of the Dutch education system assumes that all respondents complete primary education (LO). After this, they face a choice between leaving the schooling system and continuing[4]. If they opt for the latter choice, they have to choose between the 'high track' (HAVO/VWO, that is, senior general secondary education and pre-university education) and the 'low track' (LBO/MAVO, that is, junior vocational education and junior general secondary education). Once they have finished their second diploma in either track they can choose whether or not to get a third diploma, continuing with: MBO (senior secondary vocational education) if they are in the low track, or HBO/WO (higher professional education and university) if they are in the high track.

[4]Since I measure education as the highest finished level of education, continuing education actually means continuing *and* finishing a subsequent level of eduction. Even though continuing education after primary education was compulsory during almost the entire historical period that is being studied, finishing a subsequent level of education was not compulsory.

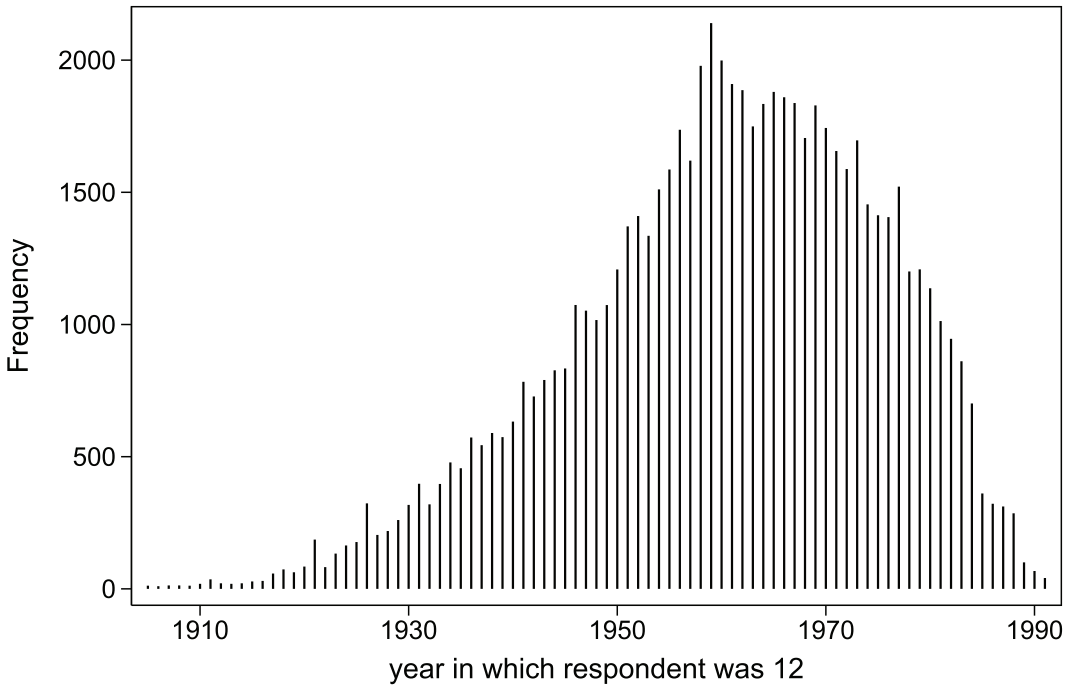
Figure 6.3: Cohorts covered by each survey (survey numbers refer to the data references)

6.4.2 The data

The data were obtained from the International Stratification and Mobility File (ISMF) (Ganzeboom and Treiman, 2009). The ISMF now contains 55 surveys on the Netherland, carried out between 1958 and 2006. These were merged to increase the time period covered and the number of respondents, and to lessen the effect of individual surveys' idiosyncrasies. The cohorts covered by each survey are represented in Figure 6.3. It shows that information on the earliest and most recent cohorts primarily originates from a few surveys, while information on the middle cohorts originates from many surveys.

The purpose of this analysis is to compare the effect of a family's SES on the highest achieved level of education and on probabilities of passing transitions, both between men and women and across cohorts. Time was measured by the year in which the respondent was 12, scaled in decades since 1910. Information was available for the cohorts born between 1905 and 1991. Cohort is allowed to have a non-linear effect by representing it as a restricted cubic spline (Harrell, 2001; Royston and Parmar, 2002) as implemented in Stata (StataCorp, 2007) as the `mkspline` command. A restricted cubic spline means that the variable is split up at a minimum of three points

Figure 6.4: Number of observations per cohort



(or knots); in this case, cohort is split up at: 1920, 1950 and 1980. Between the first and the last knot the trend is represented by a cubic spline and before the first and after the last knot the trend is restricted to be linear. This restriction leads to a relatively stable non-linear curve. A family's SES was measured according to the father's score on the International Socio-Economic Index (ISEI) of occupational status (Ganzeboom and Treiman, 2003), as this measure was available for the largest number of cohorts. The original ISEI score is a continuous variable ranging from 10 to 90, but it was standardized to have a mean of 0 and a standard deviation of 1 for the cohort born in 1960 (approximately the cohort with the most observations in this study). Survey weights were used where available. The weighted number of respondents was 82,384, and after removing respondents with missing observations on any of the variables, 71,141 respondents remained.[5] The number of respondents was unequally distributed over the cohorts, as is shown in Figure 6.4.

A scale for the level of education was needed in order to estimate the relationship between the IEOpps and IEOut using equation (6.3). The scale that will be used in this example is similar to the one estimated in Chapter 3, which is estimated in such a

[5] Various Multiple Imputation models (Little and Rubin, 2002) were tried in Chapter 4 of this dissertation and none of them caused the conclusions to be changed.

way that it maximized the direct effect of education on income while controlling for the father's occupational status. This scale does not change over time, as I established in that chapter that even though the effect of education on occupational status changed over time, the scale of education remained constant. However, if evidence was found that the scale of education also changed over time, then such a changing scale could have easily been incorporated in the decomposition. For interpretability, the scale was coded in such a way that the mean was 0 and the variance was 1 for the cohort born in 1960.

6.4.3 Generalizing the decomposition to a tracked system

The model for the Dutch educational system as represented by Figure 6.2 is more complicated than the model in Figure 6.1, which was used to illustrate the decomposition of IEOut into IEOpps and weights. Whereas the model used in the example consists of a sequence of decisions to either continue or to stop, the model for the Dutch system also contains a 'branching point', or a choice between tracks. In this sense the model is akin to those proposed by Lucas (2001) and Breen and Jonsson (2000). This raises the question of whether the decomposition still holds in the more complicated model. For that reason the decomposition is derived again for the more complicated model. As before, logistic regressions were used to model the probabilities of passing the different transitions. Again, the IEOpp and the predicted probabilities belonging to transition k are represented by λ_k and $\hat{p}_{ki}$ respectively. The predicted level of education is now represented by equation (6.4).

$$
\begin{aligned}
E(L_i) \quad = \quad & (1-\hat{p}_{1i})l_1 + \\
& \hat{p}_{1i}(1-\hat{p}_{2i})(1-\hat{p}_{3i})l_2 + \\
& \hat{p}_{1i}(1-\hat{p}_{2i})\hat{p}_{3i}l_3 + \\
& p_{1i}\hat{p}_{2i}(1-\hat{p}_{4i})l_4 + \\
& p_{1i}\hat{p}_{2i}\hat{p}_{4i}l_5
\end{aligned}
\tag{6.4}
$$

Recall that the IEOut is first derivative of equation (6.4) with respect to a family's SES. This derivative is shown in equation (6.5).

$$\begin{array}{llllllll}
\frac{\partial E(L_i)}{\partial SES} & = & & & & & & \\
\{1 & \times & \hat{p}_{1i}(1-\hat{p}_{1i}) & \times & [(1-\hat{p}_{2i})(1-\hat{p}_{3i})l_2+ & & & \\
 & & & & (1-\hat{p}_{2i})\hat{p}_{3i}l_3+ & & & \\
 & & & & \hat{p}_{2i}(1-\hat{p}_{4i})l_4+ & & & \\
 & & & & \hat{p}_{2i}\hat{p}_{4i}l_5-l_1] & \} & \lambda_1 & + \\
\{\hat{p}_{1i} & \times & \hat{p}_{2i}(1-\hat{p}_{2i}) & \times & [(1-\hat{p}_{4i})l_4+\hat{p}_{4i}l_5- & & & \\
 & & & & (1-\hat{p}_{3i})l_2-\hat{p}_{3i}l_3] & \} & \lambda_2 & + \\
\{\hat{p}_{1i}(1-\hat{p}_{2i}) & \times & \hat{p}_{3i}(1-\hat{p}_{3i}) & \times & [(l_3-l_2)] & \} & \lambda_3 & + \\
\{\hat{p}_{1i}\hat{p}_{2i} & \times & \hat{p}_{4i}(1-\hat{p}_{4i}) & \times & [(l_5-l_4)] & \} & \lambda_4 &
\end{array} \tag{6.5}$$

Just as with the example described in section 6.3, IEOut is a weighted sum of the IEOpps, the λ_ks. The weights (the parts between curly brackets) consist of the same three parts:

1. The proportion of people at risk (1, $\hat{p}_{1i}$, $\hat{p}_{1i}(1-\hat{p}_{2i})$, and $\hat{p}_{1i}\hat{p}_{2i}$ respecively).
2. A part ($\hat{p}_{ki}(1-\hat{p}_{ki})$) that is small if virtually everybody passes or fails that transition and is largest when the probability of passing is 0.5.
3. The differences between the expected levels of education of those who pass the transitions and those who do not (these are the parts in the square brackets).

This case illustrates that the relationship between IEOut and IEOpp can be extended to tracked education systems. Using the same logic, the result can be extended to even more complex systems, such as those with more than two tracks. In that case a multinomial logit would be used to estimate the IEOpp. The Stata (StataCorp, 2007) package `seqlogit` (Buis, 2007b), which implements the decomposition, applies to this general version of the sequential logit model. The only limitation is that if one uses data with only the highest achieved level of education, one must ensure that for these more complicated systems, each level can only be reached through one — and only one — path through the education system.

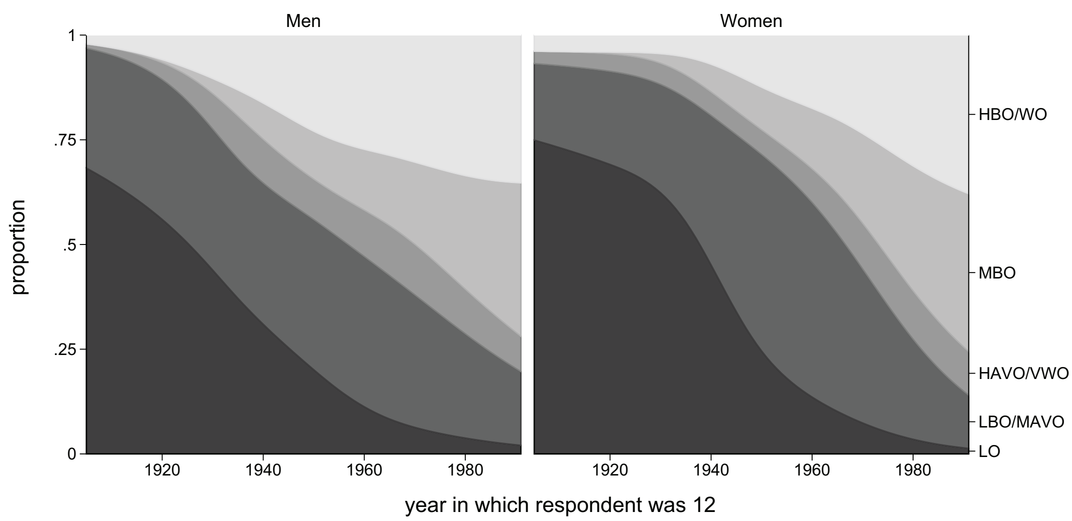
6.4.4 Results

The following analysis consists of three parts. First, a descriptive analysis is performed on the differences in transition probabilities between men and women, and between cohorts. Second, the sequential response model described in the previous section is estimated. The results from this model are used to compute the IEOpps, the weights and the IEOut. Together these provide a detailed picture of status educational inequality and how it is influenced by educational expansion and gender inequality.

Third, the relationship between the transition probabilities and the weights is investigated in more detail by looking at the three components of the weights: the proportion at risk, the closeness of the transition probability to 50%, and the expected increase in the level of education when passing a transition.

The distribution of the highest achieved level of education is shown in Figure 6.5, for both males and females and for different cohorts. The changes over cohorts were smoothed using the `proprcspline` package (Buis, 2009a) in Stata (StataCorp, 2007). As with most other countries, the Netherlands experienced a period of educational expansion during the twentieth century. The proportion of pupils who only achieved LO (primary education) dropped dramatically, while the proportion attaining HBO/WO (higher professional and university) education and MBO (higher secondary vocational) strongly increased. Figure 6.5 also shows that MBO is a recent level of education. Whereas no one from the earlier cohorts completed this level of education, MBO completion has rapidly grown to about 40%. Furthermore, women experienced all of these developments later than men.

Figure 6.5: Distribution of highest achieved level of education for men and women over cohorts



To investigate the IEOpps and IEOut and how they are influenced by gender and educational expansion (differences in the distribution of education between men and women and between cohorts respectively), sequential logit models were estimated separately for both men and women. The other variables are: cohort measured as a restricted cubic spline with knots at 1920, 1950, and 1980; the father's occupational status; and an interaction term with cohort. A model with a non-linear interaction between the father's occupational status and cohort was also estimated using the same restricted cubic spline as the main effect of cohort, but the non-linear terms proved to be non-significant (χ^2=4.73 with 4 *df* for men and χ^2=5.50 with 4 *df* for women). The results of this model are shown in Tables 6.1 and 6.2. The effects are log-odds ratios. The main effects of the father's occupational status are the IEOpps for the cohort born in 1910. This shows that the IEOpps for the higher transitions (in particular LBO/MAVO versus MBO and HAVO/VWO versus HBO/WO) are smaller than for the the lower transitions. This pattern has also been found by many other studies using sequential response models (Mare, 1980; Shavit and Blossfeld, 1993). Two explanations are commonly given for this phenomenon. First, persons passing the higher transitions are on average older than persons passing the lower transitions, and older persons are less likely to be influenced by their parents than younger persons (Shavit and Blossfeld, 1993). Second, selection on unobserved variables is likely to induce a negative correlation between the observed and unobserved variables, thus suppressing the effect of the observed variables at the higher transitions (Mare, 1981) (although Cameron and Heckman (1998) show that this does not always have to be the case). The interaction terms represent the change in effect for every ten-year change in cohort. These show that the effect of the father's occupational status changed most for the first transition. For men, this is the only transition in which the IEOpp changed significantly over cohorts. This pattern has already been found in the Netherlands (De Graaf and Ganzeboom, 1993), and is being found more frequently in studies of other countries (Breen and Jonsson, 2005).

From these results, one can derive predicted levels of education for each level of the father's occupational status, forming a non-linear regression line. Figure 6.6 presents these lines for three cohorts (1910, 1950, and 1990), and for men and women. The slope of this regression line will reveal how much the expected level of education changes when the father's occupational status changes by one unit, thus providing the IEOut. This slope is evaluated at the average father's occupational status. The father's occupational status is standardized, so a respondent with a typical background has a father's status of 0[6]. This figure shows that in all cases, having a father with a higher socioeconomic status will lead to a higher expected level of education. Also, it shows that while women initially suffered a disadvantage, they have overtaken men in the most recent cohort. Finally, the results show that for the earliest cohort, the inequality of educational outcomes for a respondent with a typical background was relatively

[6]However, the standardization uses the cohort born in 1960, and the average of the father's status increased over cohorts. The average of father's occupational status remained reasonably constant until about 1930 at about -0.2 and then steadily increased to 0.5. These changes not only reflect changes in economic structure, but also changes in the difference in the number of respondents between higher and lower status fathers. Consequently, it is hard to give a substantive interpretation to these changes. To simplify the analysis, a respondent with a typical background will be fixed at the typical background (average father's occupational status) for a typical cohort (1960).

Table 6.1: Sequential response model for men

	LO v more	LBO/MAVO v HAVO/VWO	LBO/MAVO v MBO	HAVO/VWO v HBO/WO
Father's status	0.912	0.694	0.263	0.446
	(15.28)	(14.19)	(3.44)	(5.91)
Father's status X of Cohort	-0.068	-0.015	-0.004	-0.033
	(-5.09)	(-1.62)	(-0.30)	(-2.35)
RC spline term 1 of Cohort	0.566	0.316	0.461	0.461
	(17.54)	(9.15)	(9.45)	(7.93)
RC spline term 2 of Cohort	-0.000	0.013	0.002	0.015
	(-0.01)	(7.08)	(0.97)	(4.82)
Constant	-0.590	-1.470	-2.893	-0.806
	(-6.36)	(-13.13)	(-18.00)	(-4.24)
N	43770			
Log likelihood	-50032.082			

z statistics in parentheses

Table 6.2: Sequential response model for women

	LO v more	LBO/MAVO v HAVO/VWO	LBO/MAVO v MBO	HAVO/VWO v HBO/WO
Father's status	0.874	1.021	0.412	0.079
	(15.33)	(17.23)	(5.21)	(0.88)
Father's status X cohort	-0.068	-0.063	-0.021	0.029
	(-5.34)	(-6.00)	(-1.51)	(1.82)
RC spline term 1 of Cohort	0.743	0.103	0.129	0.345
	(21.26)	(2.27)	(2.33)	(4.67)
RC spline term 2 of Cohort	-0.001	-0.008	-0.022	0.008
	(-0.24)	(-3.58)	(-8.27)	(2.36)
Constant	-1.727	-1.693	-2.431	-0.760
	(-17.05)	(-10.88)	(-12.87)	(-2.99)
N	43675			
Log likelihood	-45830.33			

z statistics in parentheses

small (the curve is rather flat), because everybody in the immediate neighbourhood of the respondent with an average family background had an expected level of education that was close to the minimum. However, in this same cohort, respondents with very high-status parents do a lot better than the other respondents, which would lead to a high inequality of educational outcome. In other words, in this chapter estimates of the local educational inequality will be obtained, and if one were to estimate a measure of global educational inequality instead, the estimate would be higher for the earliest cohorts.

Figure 6.6: Expected highest achieved level of education according to the sequential logit model

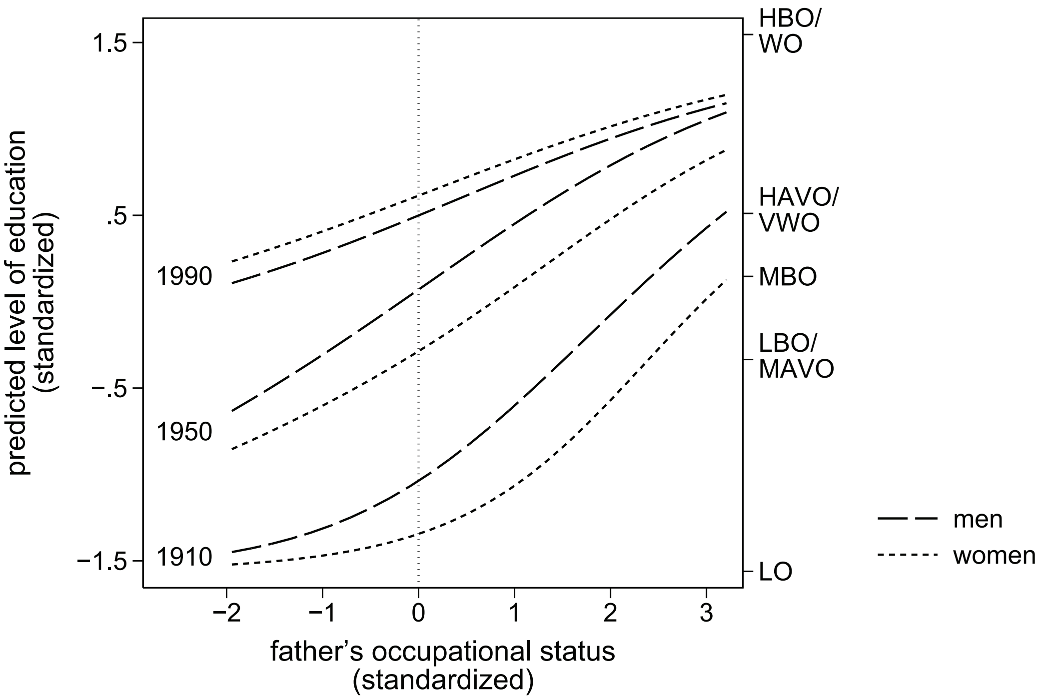
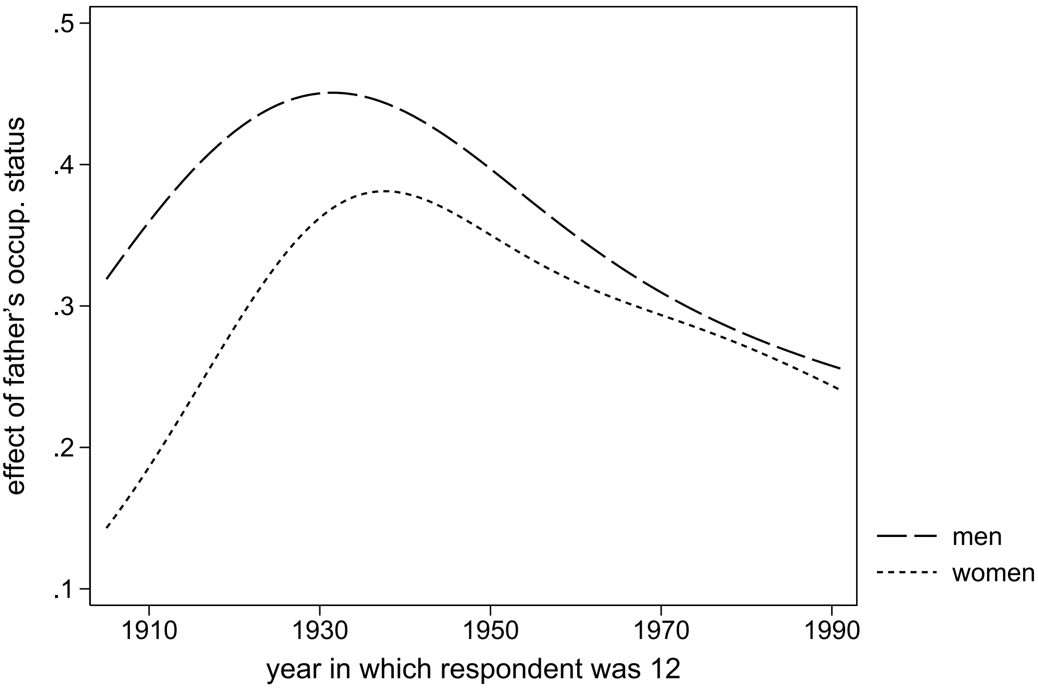


Figure 6.7 shows the estimates of IEOut that have been derived from the sequential logit model. Both education and the father's occupational status are scaled in such a way that the mean for the cohort 1960 is 0 and the standard deviation is 1. So this measure of IEOut is similar to a standardized regression coefficient. IEOut displays two striking features: the first is the trend in IEOut, which initially increases and then decreases. The second feature is the initially lower IEOut for women. These are not unique to the sequential logit model, since in Chapter 4 I found similar patterns using different methods. In order to explain these patterns IEOut will be broken down into its components, in three steps.

Figure 6.7: IEOut according to the sequential logit model



The first step looks at the contributions of each transition to IEOut. The IEOut is a weighted sum of each transition's IEOpp, so each transition contributes the amount of weight times IEOpp to IEOut. This is shown in Figure 6.8. A striking feature is that the final two transitions (HAVO/VWO to HBO/WO and LBO/MAVO to MBO) contribute negligible amounts to IEOut. Furthermore, the initial increase and later decrease in IEOut seems to be primarily the result of what happened at the first transition. Finally, there has been a shift between the first and the second transitions as the dominant source of IEOut.

The second step consists of breaking up each transition's contribution into its two parts: the weight and the IEOpp. Since the contribution is the product of these two terms, it can be visualized as the area of a rectangle, with a height equal to the IEOpp and a width equal to the weight. For men and women, this is shown in Figures 6.9 and 6.10. The horizontal axis shows the weights and the vertical axis the IEOpp, while the columns represent the cohorts and the rows represent the transitions. These figures show that the initial increase in the contribution of the first transition is due to an increase in its weight, while the later decrease of this transition is due to both a decrease in the weight and a decrease in the IEOpp. The increase in importance of the second transition is entirely due to the increase in the weight of this transition.

Figure 6.8: Contribution of each transition to IEOut

For women, this increase in weight actually offsets a decrease in IEOpp. The low contributions of both higher transitions are due to both low IEOpp and low weight.

The third step breaks the weights down into their three components. Figure 6.11 (a) shows the changes in the weights over time in more detail. The changes in these weights capture the consequences of changes in the distribution of education on IEOut. These weights are the product of three components: the proportion of people at risk at each transition (Figure 6.11 (b)); the closeness to 50% of the proportion of people passing (the variance) (Figure 6.11 (c)); and the difference in the expected level of education between those passing and those failing a transition (Figure 6.11 (d)). Figure 6.11 shows that the initial increase and the later decline in the first transition's influence is primarily due to the variance. Initially, any inequality at the first transition affected few people, because a low proportion passed. As the proportion of people passing increased, the transition received more weight, until half of the students passed, after which inequality affected less people again because few people failed. The increase in importance of the second transition is partly due to the variance, but also to a strong increase in the number of students that are at risk of making this transition. Notice that these developments at the first two transitions provide a substantively interpretable mechanism through which educational expansion influences IEOut. For women, these developments have occurred later, leading initially to smaller weights. The last two transitions receive relatively small weights because relatively few people are at risk of passing these transitions, and those who pass gain relatively little. Those who pass the first two transitions gain both the immediate increase in level of education and the possibility of gaining an extra level of education (either MBO or

HBO/WO), while in the third and fourth transition, people gain only the immediate increase in level of education.

Figure 6.9: Decomposition of IEOut into IEOpps and weights

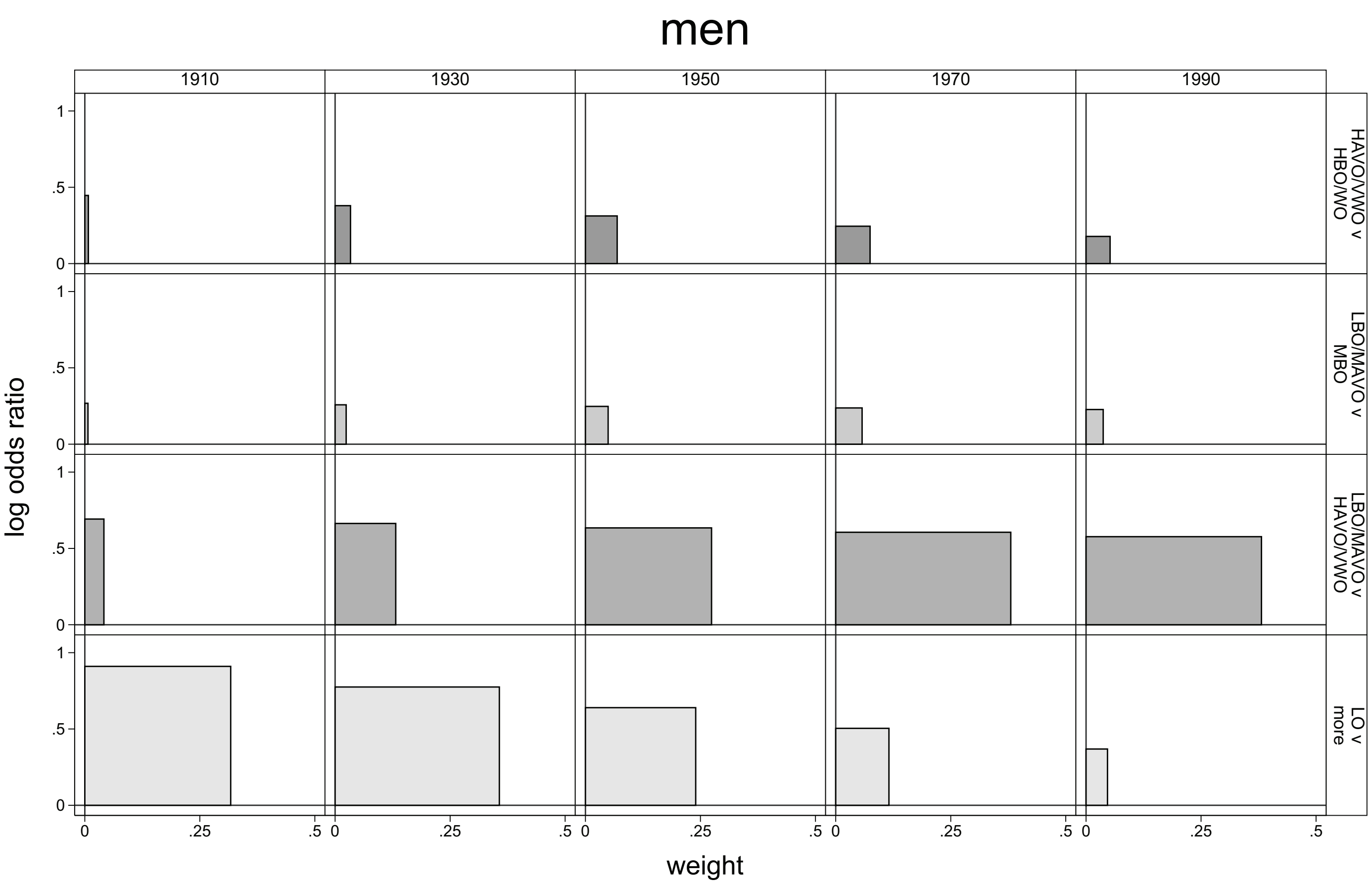


Figure 6.10: Decomposition of IEOut into IEOpps and weights

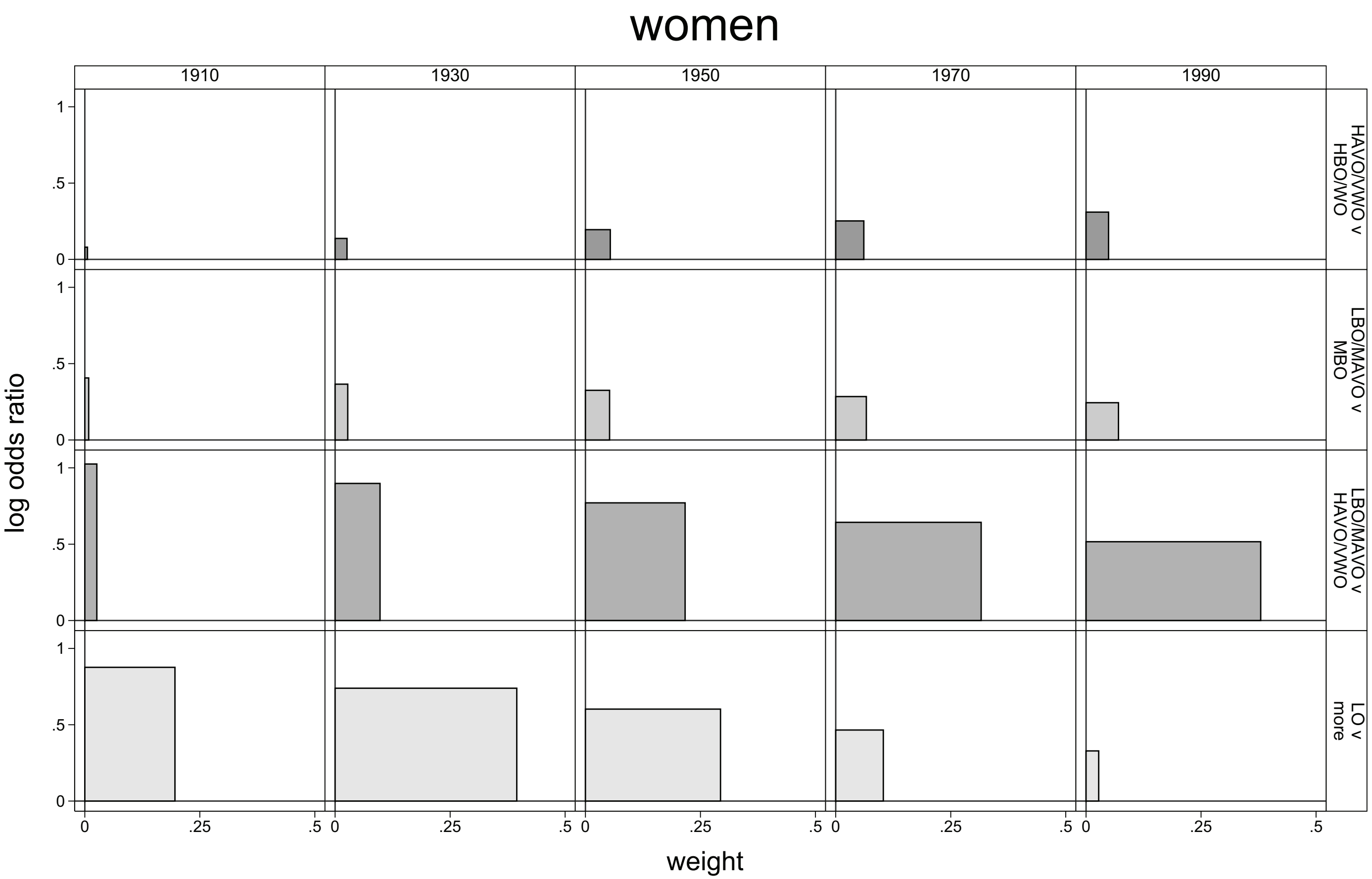


Figure 6.11: Weights and their components

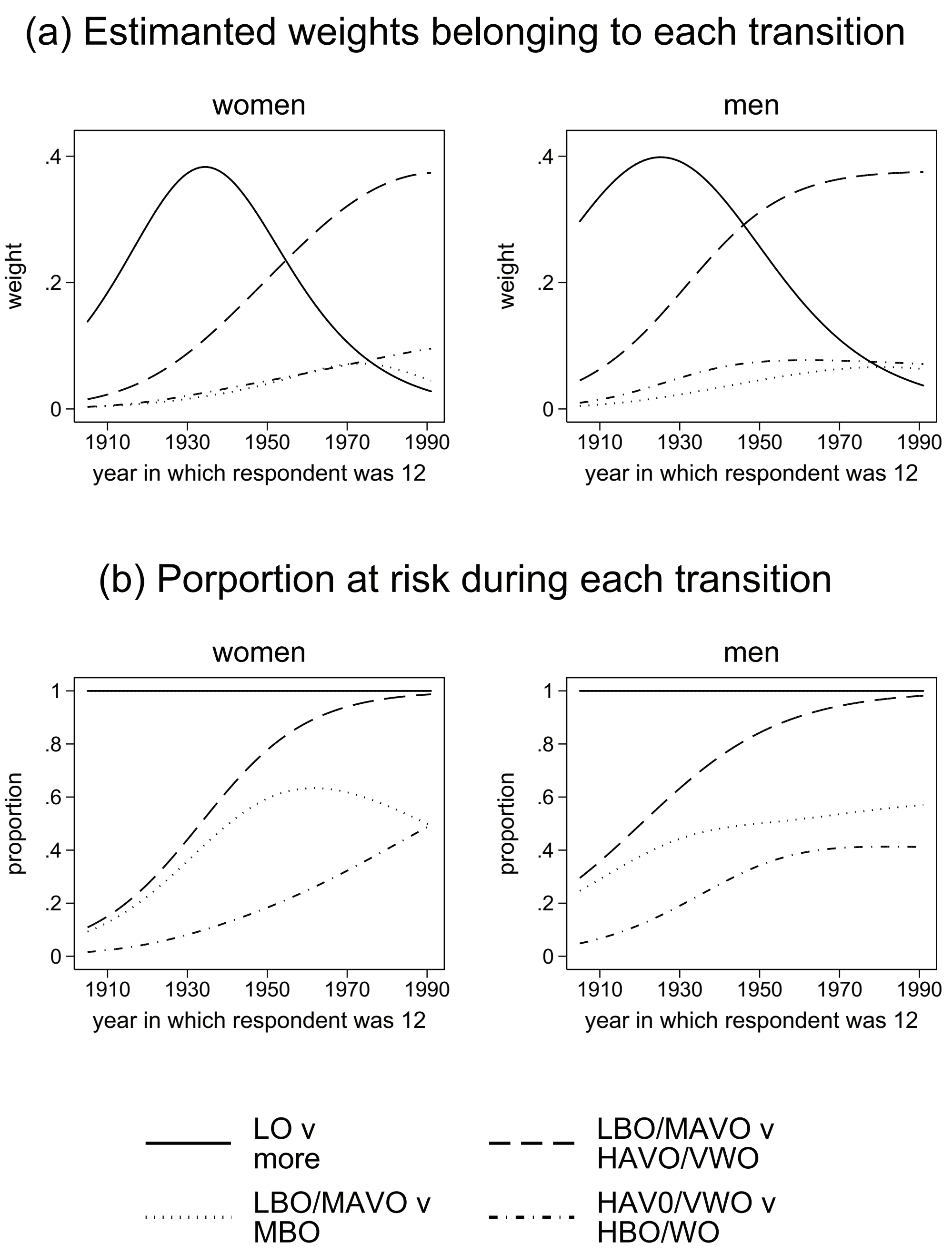
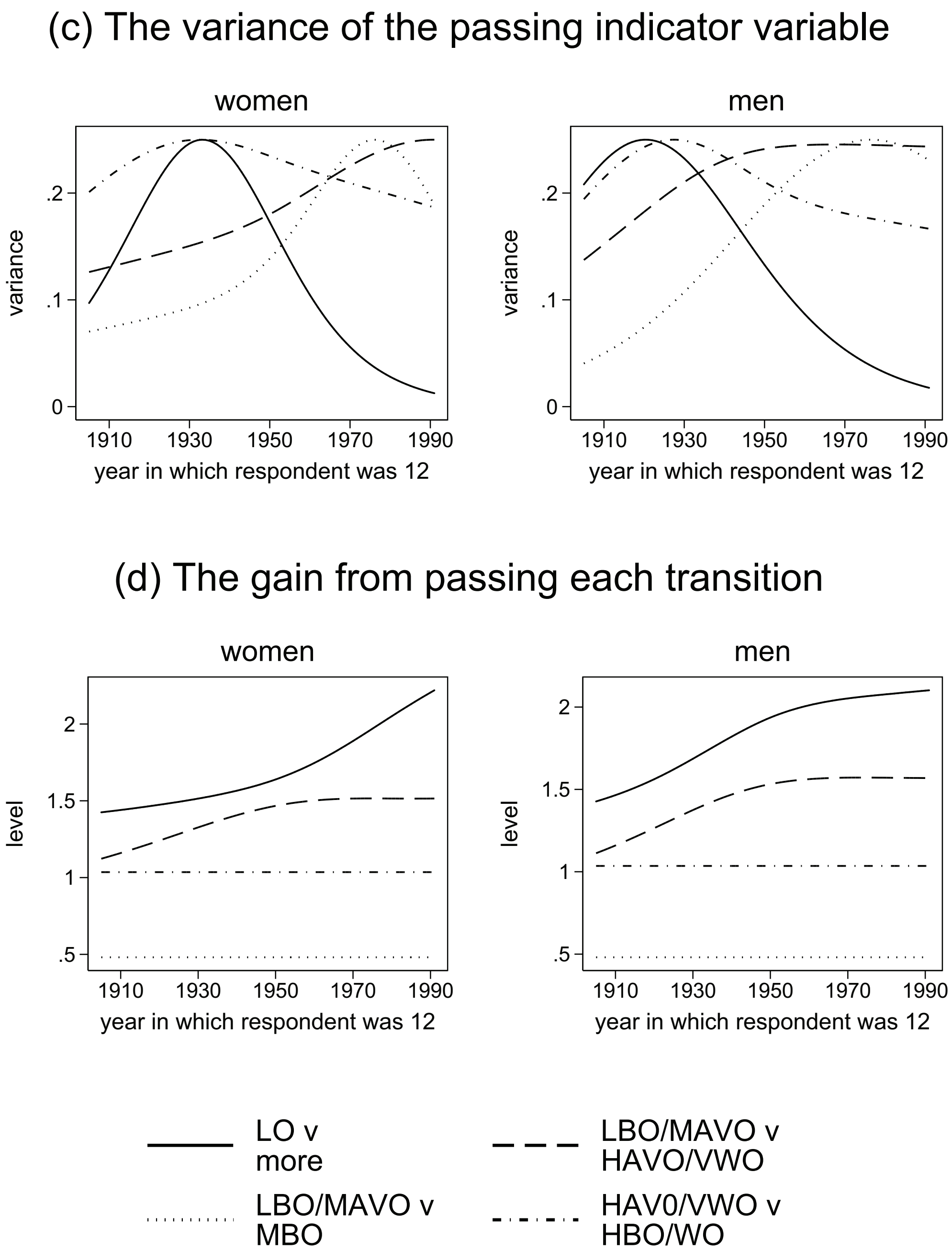


Figure 6.11: Weights and their components (continued)



6.5 Conclusion

This chapter began by making a distinction between two types of inequality of educational opportunity (IEO): inequality of educational opportunities during the process of attaining education, which I called Inequality of Educational Opportunities proper (IEOpp), and inequality of educational opportunities in terms of the outcome of the educational process, which I called Inequality of Educational Outcomes (IEOut). Mare (1981) demonstrated that differences in IEOut across cohorts (or other groups) depend on both the differences in IEOpp and differences in the distribution of education. However, this literature did not study the relationship between IEOpp, IEOut and the distribution of education, but instead treated this relationship as a 'black box'. This was used as an argument for studying only IEOpps and for controlling for the distribution of education rather than of studying its effects. This chapter seeks to change this by answering the following two questions:

- How are IEOut and IEOpp related to one another, and how can this relation be used for a meaningfully integrated analysis of IEOpp and IEOut?
- How are IEOut and the distribution of education related to one another, and how can this relation be used for an analysis of the influence of changes in the distribution of education on IEOut?

The first question is based on the observation that IEOpp and IEOut are not competing descriptions of IEO but natural complements, because a description of a process (the IEOpps) and a description of the outcome of that process (the IEOut) are natural complements. Treating IEOpps and IEOut as complementary creates the challenge to move beyond a separate discussion of these two estimates to an integrated discussion of IEOpp and IEOut. The second question is based on the observation that the influence of changes in the distribution of education on estimates of IEO is a phenomenon of substantive interest. One such change in the distribution of education is the general increase in highest achieved level of education over cohorts, which is one of the most universal and far-reaching changes in educational systems across countries during the 20^{th} century (Hout and DiPrete, 2006). The consequences for IEO of such a major change in the educational system deserve to be studied rather than just controlled for.

These questions are answered by showing that the sequential logit model, which was proposed by Mare (1981) for estimating IEOpps, also implies an estimate for IEOut. This estimate of IEOut is a weighted sum of IEOpps such that an IEOpp that belongs to a certain transition between levels of education receives more weight if more people are at risk of passing that transition; if passing or failing the transition is less universal (that is, if the proportion of respondents who pass is closer to 50%); and if there is a larger difference in the expected level of education between people who pass and fail that transition. This decomposition shows how IEOpp and IEOut are related and allows for an integrated discussion of these two by showing to what extent each transition's IEOpp contribute to IEOut. The weights also allows one to study the impact of changes in the distribution of education on IEOut, as these weights depend on the distribution in a substantively interpretable way.

The application of this decomposition was illustrated using an analysis of changes in IEO in the Netherlands between 1905 and 1991. It showed that the composition of IEOut shifted from being primarily determined by the IEOpp of the first transition (whether or not to continue after primary education) to being primarily determined by the IEOpp of the second transition (the choice between the vocational and the academic track). The IEOpps of the later transitions contributed relatively little to IEOut throughout the period being studied. The differences in the distribution of education across cohorts (educational expansion) and gender (gender educational inequality) were shown to explain this shift in importance between the first and second transitions and two main features of the trend in IEOut. First, the trend over cohorts showed an initial increase followed by a decrease. Second, the IEOut is initially lower for women. The initial increase in IEOut can be explained by the increase in the proportion of students that pass the first two transitions from less than 50% to around 50%, thus initially increasing the weights for both transitions. The weight for the second transition also increased as more students became at risk of passing that transition. The subsequent decrease in IEOut happened because the weight of the first transition's IEOpp sharply decreased since passing that transition became near universal. These changes also explain the shift in importance between the IEOpps of the first and second transitions. The decrease in the difference between men and women in IEOut was caused by the fact that initially fewer women passed each transition, causing each transition's weight to be less for women than for men. For the later cohorts, weights were approximately equal between men and women, because women were as likely as men —or even more likely — to pass transitions, thus causing a convergence in IEOut of men and women.

This chapter defined IEOut in such a way that it is meaningfully influenced by changes in the distribution of education. There is however an important body of research in this literature that uses log-linear models that summarize the IEOut in a single odds ratio (De Graaf and Ganzeboom, 1990; Ganzeboom and Luijkx, 2004a,b). Unlike the measure of IEOut used in this chapter, the odds ratio controls changes in the distribution of education, that is, educational expansion. I would argue that this is not necessarily a good thing: changes in IEOut over time are studied not because we think that time directly influences IEOut, but that society changes over time and these changes lead to changes in IEOut. The aim of such an analysis should be to study how these changes in society influenced IEOut, not sweep them under the carpet by controlling for them.

In future research, the decomposition presented in this chapter can be generalized in a number of ways. First, the decomposition can be applied to some models that have been proposed to address the critique on the sequential logit model by Cameron and Heckman (1998). The decomposition can be applied to those models that are direct adaptations of the sequential logit model (for example: Mare 1993, 1994; and Chapter 7 of this dissertation), but not to models that do not use the (multinomial) logit link function (for example Lucas et al., 2007; Holm and Jæger, 2008). Second, the decomposition requires that each level of education is assigned a value. In this chapter, these values are constant over time, but there has been debate on whether the values of educational categories have changed as a consequence of strong changes in the distribution of education and the labor market (Rumberger, 1981; Clogg and Shockey, 1984; Groot and Maassen van den Brink, 2000). If one has time-varying estimates of the value of the levels of education, then these could also be incorporated in the decomposition. Changes in these values would influence IEOut through only one of the three components of the weight: the difference in the expected highest achieved level of education between people who pass and fail a transition. The decomposition could thus also be used to study the impact of possible changes in the values of educational levels. Third, the analysis is based on data on the highest achieved level of education in combination with a stylized model of the education system. The transitions that respondents have passed were derived from these two pieces of information rather than being directly observed. The main advantage of using highest achieved levels of education is that much more data is available on the highest achieved level of education and that this data covers a larger period than data on actual transitions. However, an additional analysis using observed transitions is desirable. An interesting question that could be answered this way would be the impact of 'second chance paths', that is, paths where one switches from one track to another. The effect of these second chance paths on IEO is not clear: on the one hand these second chance paths could offer a way out of lower tracks for those disadvantaged students that were disproportionably assigned to them, on the other hand students from advantaged background are generally better capable of making the best use of these 'loopholes'. An additional advantage of using observed transitions is that one no longer has to rely on pseudo-cohorts to measure trends over time, as in that case one directly observes when a transition occurred.

In conclusion, this chapter has shown how the study of educational inequality can be enriched by studying IEOpp and IEOut as complementary pieces of information and by studying the impact of the distribution of education, rather than by simply controlling for it. This has the key advantage of enabling an integrated discussion of IEOpp and IEOut and the study of the impact of phenomena such as educational expansion.

Appendix: Derivation of equation (6.3)

Equation (6.3) is the first derivative of equation (6.2). Equation (6.2) is repeated below:

$$E(L_i) = (1 - \hat{p}_{1i})l_0 + \hat{p}_{1i}(1 - \hat{p}_{2i})l_1 + \hat{p}_{1i}\hat{p}_{2i}(1 - \hat{p}_{3i})l_2 + \hat{p}_{1i}\hat{p}_{2i}\hat{p}_{3i}l_3$$

whereby the $\hat{p}_{ki}$s are represented by equation (6.1), repeated below:

$$\hat{p}_{ki} = \frac{\exp(\alpha_k + \lambda_k SES_i)}{1 + \exp(\alpha_k + \lambda_k SES_i)} \quad \text{if} \quad y_{k-1\,i} = 1$$

This derivative can be computed using the sum rule,[7] the product rule,[8], and the derivative of a logistic regression equation.[9] Using the sum rule, the first derivative can be written as:

[7]Suppose that we have two functions of SES: $f(SES)$ and $g(SES)$. The sum rule states that the derivative of the sum of these functions with respect to SES is (e.g. Gill, 2006, p. 190):

$$\frac{\partial(f(SES) + g(SES))}{\partial SES} = \frac{\partial f(SES)}{\partial SES} + \frac{\partial g(SES)}{\partial SES}$$

[8]The product rule states that the derivative of the product of these functions with respect to SES is (e.g. Gill, 2006, p. 191):

$$\frac{\partial(f(SES) \times g(SES))}{\partial SES} = \frac{\partial f(SES)}{\partial SES} g(SES) + \frac{\partial g(SES)}{\partial SES} f(SES)$$

A special case occurs when a function of SES is multiplied by a constant c because the first derivative of a constant is zero:

$$\frac{\partial(cf(SES))}{\partial SES} = \frac{\partial f(SES)}{\partial SES} c + \frac{\partial c}{\partial SES} f(SES) = \frac{\partial f(SES)}{\partial SES} c$$

[9]Equation (6.1) is a logistic regression equation, which has a known first derivative (e.g. equation 3.14 Long, 1997):

$$\frac{\partial \hat{p}_{ki}}{\partial SES} = \hat{p}_{ki}(1 - \hat{p}_{ki})\lambda_k$$

Together with the sum and the product rule this also implies that:

$$\begin{aligned}
\frac{\partial(1 - \hat{p}_{ki})}{\partial SES} &= \frac{\partial 1}{\partial SES} + \frac{\partial - \hat{p}_{ki}}{SES} \quad \text{(sum rule)} \\
&= -\frac{\partial \hat{p}_{ki}}{SES} \quad \text{(product rule)} \\
&= -\hat{p}_{ki}(1 - \hat{p}_{ki})\lambda_k
\end{aligned}$$

$$\frac{\partial E(L_i)}{\partial SES} = \frac{\partial(1-\hat{p}_{1i})l_0}{\partial SES} + \frac{\partial p_1(1-\hat{p}_{2i})l_1}{\partial SES} + \frac{\partial \hat{p}_{1i}\hat{p}_{2i}(1-\hat{p}_{3i})l_2}{\partial SES} + \frac{\partial \hat{p}_{1i}\hat{p}_{2i}\hat{p}_{3i}l_3}{\partial SES}$$

Using the product rule, this can be rewritten as:

$$\begin{aligned}
\frac{\partial E(L_i)}{\partial SES} = {} & l_0 \frac{\partial(1-\hat{p}_{1i})}{\partial SES} + \\
& l_1 \left(\frac{\partial \hat{p}_{1i}}{\partial SES}(1-\hat{p}_{2i}) + \frac{\partial(1-\hat{p}_{2i})}{\partial SES}\hat{p}_{1i} \right) + \\
& l_2 \left(\frac{\partial \hat{p}_{1i}}{\partial SES}\hat{p}_{2i}(1-\hat{p}_{3i}) + \frac{\partial \hat{p}_{2i}}{\partial SES}\hat{p}_{1i}(1-\hat{p}_{3i}) + \frac{\partial(1-\hat{p}_{3i})}{\partial SES}\hat{p}_{1i}\hat{p}_{2i} \right) + \\
& l_3 \left(\frac{\partial \hat{p}_{1i}}{\partial SES}\hat{p}_{2i}\hat{p}_{3i} + \frac{\partial \hat{p}_{2i}}{\partial SES}\hat{p}_{1i}\hat{p}_{3i} + \frac{\partial \hat{p}_{3i}}{\partial SES}\hat{p}_{1i}\hat{p}_{2i} \right)
\end{aligned}$$

All derivatives in the equation are derivatives of logistic regression equations. To facilitate the comparison with the previous equation, curly brackets are used to enclose these derivatives.

$$\begin{array}{ll}
\frac{\partial E(L_i)}{\partial SES} & = \\
l_0 & \{-\hat{p}_{1i}(1-\hat{p}_{1i})\lambda_1\} + \\
l_1 & (\{\hat{p}_{1i}(1-\hat{p}_{1i})\lambda_1\}(1-\hat{p}_{2i}) + \{-\hat{p}_{2i}(1-\hat{p}_{2i})\lambda_2\}\hat{p}_{1i}) + \\
l_2 & (\{\hat{p}_{1i}(1-\hat{p}_{1i})\lambda_1\}\hat{p}_{2i}(1-\hat{p}_{3i}) + \{\hat{p}_{2i}(1-\hat{p}_{2i})\lambda_2\}\hat{p}_{1i}(1-\hat{p}_{3i}) + \\
 & \{-\hat{p}_{3i}(1-\hat{p}_{3i})\lambda_3\}\hat{p}_{1i}\hat{p}_{2i}) + \\
l_3 & (\{\hat{p}_{1i}(1-\hat{p}_{1i})\lambda_1\}\hat{p}_{2i}\hat{p}_{3i} + \{\hat{p}_{2i}(1-\hat{p}_{2i})\lambda_2\}\hat{p}_{1i}\hat{p}_{3i} + \\
 & \{\hat{p}_{3i}(1-\hat{p}_{3i})\lambda_3\}\hat{p}_{1i}\hat{p}_{2i})
\end{array}$$

The terms in this equation can be rearranged in such a way that all elements that have the same IEOpp (λ_k) in common are grouped together.

$$\begin{array}{ll}
\frac{\partial E(L_i)}{\partial SES} & = \\
\lambda_1 & \{-\hat{p}_{1i}(1-\hat{p}_{1i})l_0 + \hat{p}_{1i}(1-\hat{p}_{1i})(1-\hat{p}_{2i})l_1 + \\
 & \hat{p}_{1i}(1-\hat{p}_{1i})\hat{p}_{2i}(1-\hat{p}_{3i})l_2 + \hat{p}_{1i}(1-\hat{p}_{1i})\hat{p}_{2i}\hat{p}_{3i}l_3\} + \\
\lambda_2 & \{-\hat{p}_{2i}(1-\hat{p}_{2i})\hat{p}_1 l_1 + \hat{p}_{2i}(1-\hat{p}_{2i})\hat{p}_1(1-\hat{p}_{3i})l_2 + \\
 & \hat{p}_{2i}(1-\hat{p}_{2i})\hat{p}_{1i}\hat{p}_{3i}l_3\} + \\
\lambda_3 & \{-\hat{p}_{3i}(1-\hat{p}_{3i})\hat{p}_{1i}\hat{p}_{2i}l_2 + \hat{p}_{3i}(1-\hat{p}_{3i})\hat{p}_{1i}\hat{p}_{2i}l_3\}
\end{array}$$

Simplifying this equation will yield equation (6.3).