Direct and indirect effects in a `logit` model

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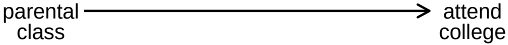
Outline

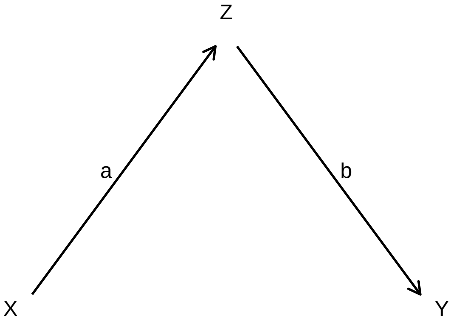
The Total Effect

$$X \longrightarrow Y$$

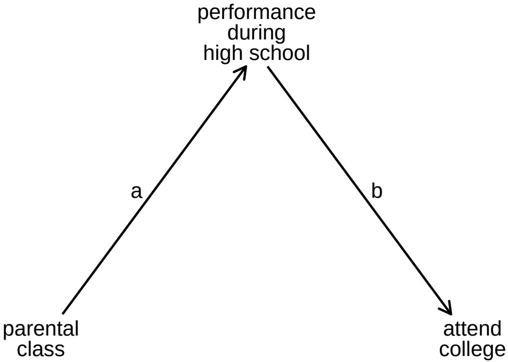
The Total Effect



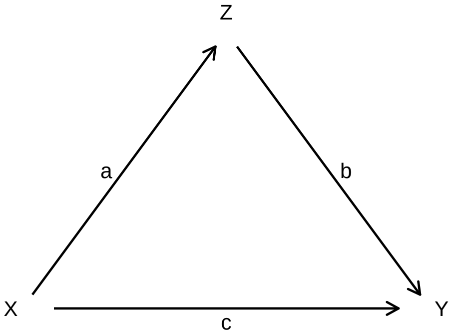
The Indirect Effect



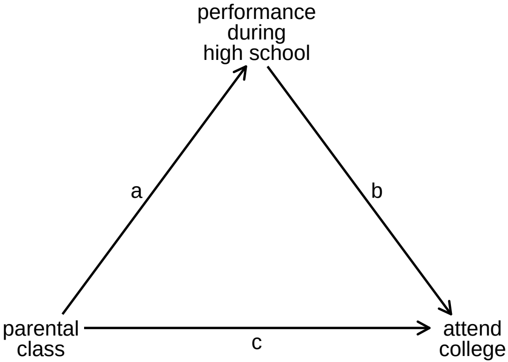
The Indirect Effect



The Direct Effect

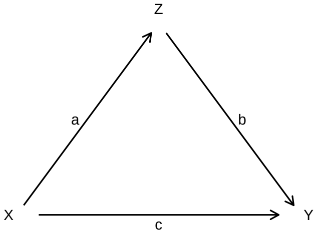


The Direct Effect



The aim

The aim is to find the size of the indirect effect relative to the total effect.



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- The direct effect is the effect of class in model 1.
- The total effect is the effect of class in model 2.
- The indirect effect is the total effect - direct effect.
- This won't work when using `logit`

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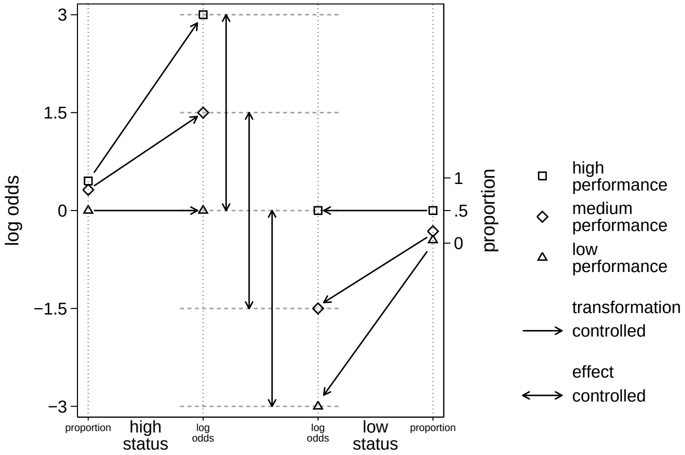
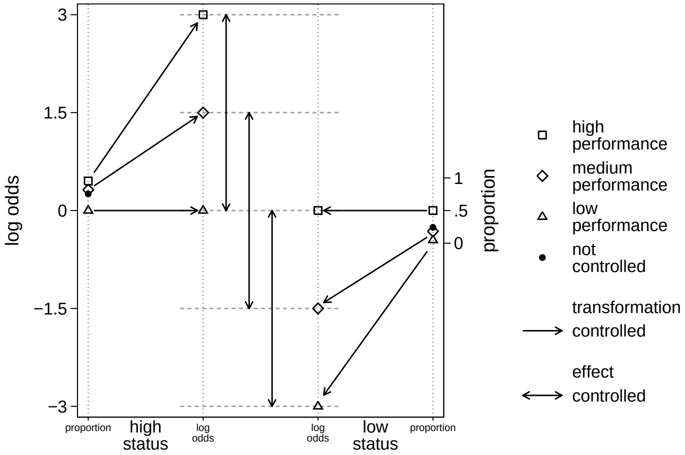
- Easiest explained when there is no indirect effect.
- The total effect = the direct effect + the indirect effect.
- So, the total effect should be the same as the direct effect when there is no indirect effect.
- So, the effect of class in a model that controls for performance (the ‘direct effect’) should be the same as the effect of class in a model that does not control for performance (the ‘total effect’).

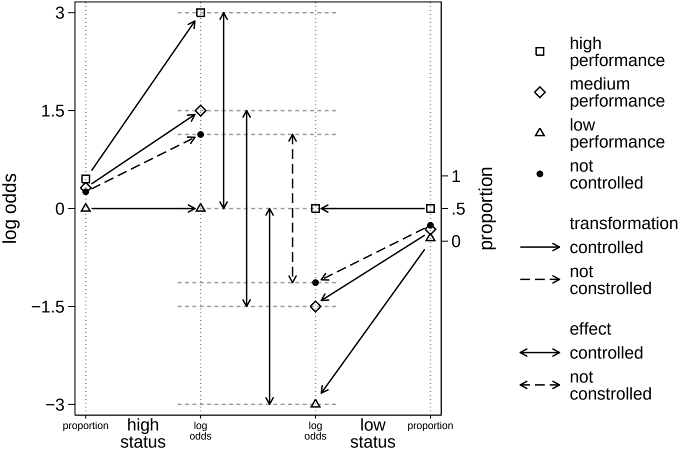
Effect while controlling for performance



Averaging the proportions



Effect while *not* controlling for performance



Outline

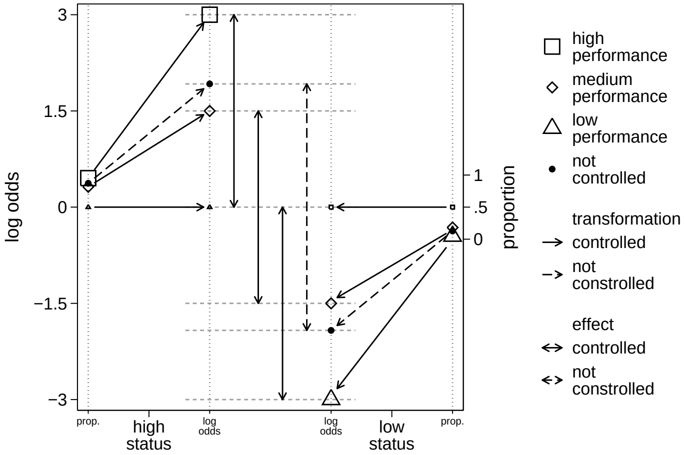
The aim

The problem

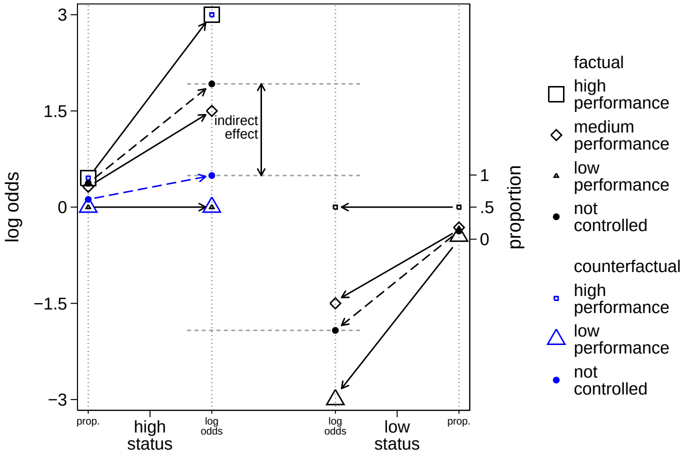
The solution

example

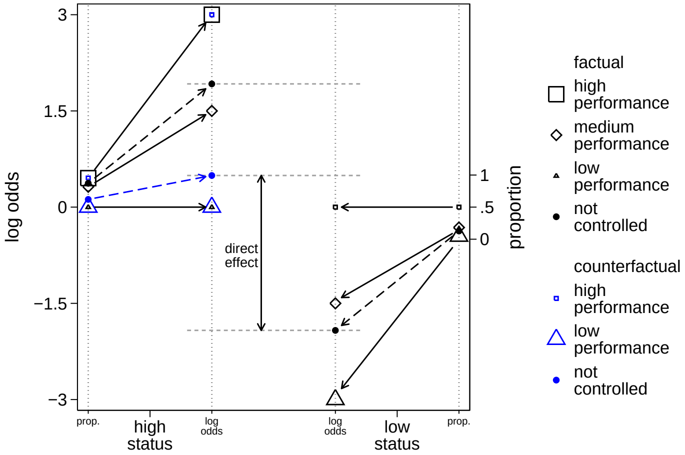
Indirect effect present



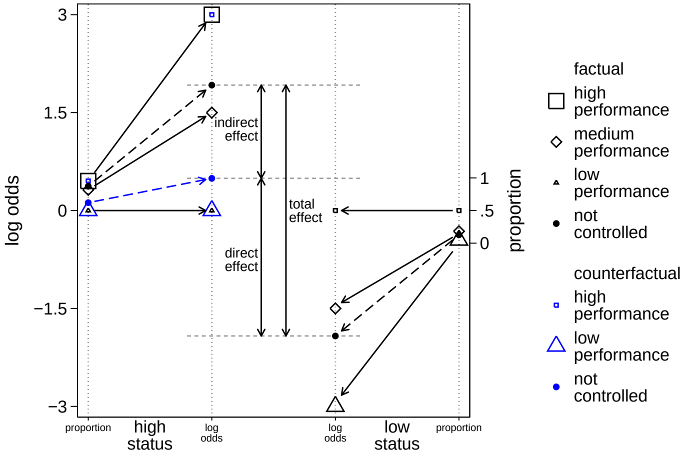
Indirect effect



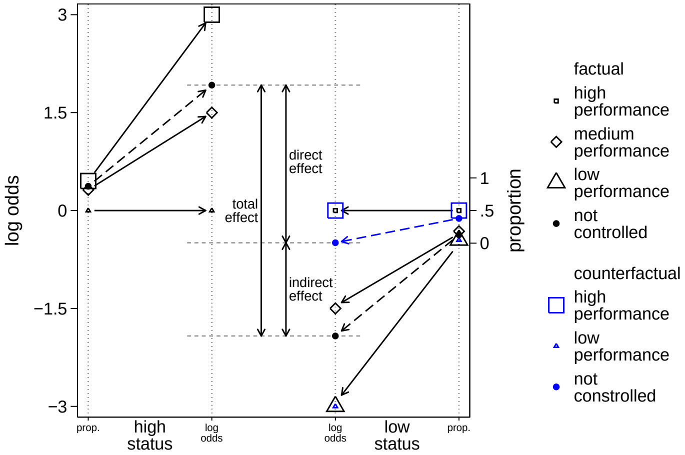
Direct effect



Direct and indirect effects in `logit`



The logic can be reversed



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- Erikson et al. (2005) propose to compute the average proportions given the observed and counterfactual distribution of performance by assuming that performance is normally distributed, and then integrate over this normal distribution.

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Extension

- Erikson et al. (2005) propose to compute the average proportions given the observed and counterfactual distribution of performance by assuming that performance is normally distributed, and then integrate over this normal distribution.
- Alternatively, these averages can be computed by predicting the observed and counterfactual proportions, add them up and divide by the number of respondents in that group.
- The latter method has the advantage of making less assumptions about the distribution of performance, as it integrates over the empirical distribution of performance instead of over a normal distribution.

Outline

Descriptives

```
. table ocf57 if !missing(hsrankq, college) , ///
> contents(mean college mean hsrankq freq)    ///
> format(%9.3g) stubwidth(15)
```

occupation of r father in 1957	mean(college)	mean(hsrankq)	Freq.
lower	.284	48.2	5,218
middle	.38	50.6	868
higher	.619	56.2	2,837

The `ldecomp` package

`ldecomp` *depvar* [*if*] [*in*] [*weight*] , `direct(`*varname*`)` `indirect(`*varlist*`)` [`obspr` `predpr` `predodds` `or` `rindirect` `normal` `range(`*##*`)` `nip(`*#*`)` `interactions` `nolegend` `nodecomp` `nobootstrap` *bootstrap_options*]

Decomposition of log odds ratios

```
. ldecomp college , direct(ocf57) indirect(hsrankq) rind nolegend
(running _ldecomp on estimation sample)

Bootstrap replications (50)
----+--- 1 ---+--- 2 ---+--- 3 ---+--- 4 ---+--- 5
..................................................    50

Bootstrap results                               Number of obs      =      8923
                                                Replications       =        50
```

	Observed Coef.	Bootstrap Std. Err.	z	P>|z|	Normal-based [95% Conf.	Interval]
2/1						
total	.4367997	.0689983	6.33	0.000	.3015655	.5720339
indirect1	.0593679	.0274903	2.16	0.031	.0054878	.1132479
direct1	.3774319	.0729045	5.18	0.000	.2345416	.5203221
indirect2	.0586611	.0277894	2.11	0.035	.0041949	.1131272
direct2	.3781386	.0731829	5.17	0.000	.2347028	.5215744
3/1						
total	1.410718	.0486595	28.99	0.000	1.315347	1.506088
indirect1	.2058881	.0176897	11.64	0.000	.1712169	.2405594
direct1	1.204829	.0471822	25.54	0.000	1.112354	1.297305
indirect2	.2012494	.0186496	10.79	0.000	.1646968	.237802
direct2	1.209468	.0472203	25.61	0.000	1.116918	1.302018
3/2						
total	.9739179	.0775048	12.57	0.000	.8220112	1.125825
indirect1	.1461109	.0303069	4.82	0.000	.0867104	.2055115
direct1	.8278069	.0730737	11.33	0.000	.684585	.9710288
indirect2	.1432144	.0317941	4.50	0.000	.080899	.2055297
direct2	.8307035	.073588	11.29	0.000	.6864737	.9749333

Relative effects

2/1r						
method1	.1359155	.0684536	1.99	0.047	.0017489	.2700821
method2	.1342975	.0691401	1.94	0.052	-.0012147	.2698096
average	.1351065	.0687727	1.96	0.049	.0003144	.2698986
3/1r						
method1	.1459457	.0121411	12.02	0.000	.1221496	.1697418
method2	.1426575	.0127593	11.18	0.000	.1176498	.1676652
average	.1443016	.0123863	11.65	0.000	.1200249	.1685782
3/2r						
method1	.1500239	.0290461	5.17	0.000	.0930946	.2069532
method2	.1470497	.0307503	4.78	0.000	.0867803	.2073192
average	.1485368	.0298296	4.98	0.000	.0900718	.2070018

Decomposition of odds ratios

```
. ldecomp college , direct(ocf57) indirect(hsrankq) or nolegend
(running _ldecomp on estimation sample)

Bootstrap replications (50)
----+--- 1 ---+--- 2 ---+--- 3 ---+--- 4 ---+--- 5
..................................................    50

Bootstrap results                               Number of obs      =      8923
                                                Replications       =        50
```

	Observed Odds Ratio	Bootstrap Std. Err.	z	P>|z|	Normal-based [95% Conf.	Interval]
2/1						
total	1.547746	.1206692	5.60	0.000	1.328423	1.80328
indirect1	1.061166	.0267866	2.35	0.019	1.009942	1.114987
direct1	1.458534	.1116674	4.93	0.000	1.2553	1.694672
indirect2	1.060416	.0264864	2.35	0.019	1.009754	1.11362
direct2	1.459565	.1118132	4.94	0.000	1.256074	1.696023
3/1						
total	4.098896	.1715291	33.71	0.000	3.776123	4.449258
indirect1	1.228616	.0194388	13.01	0.000	1.191101	1.267312
direct1	3.33619	.1467056	27.40	0.000	3.060695	3.636483
indirect2	1.22293	.0201835	12.19	0.000	1.184004	1.263136
direct2	3.351702	.1467947	27.62	0.000	3.075992	3.652124
3/2						
total	2.6483	.2089437	12.34	0.000	2.26887	3.091182
indirect1	1.157325	.031425	5.38	0.000	1.097343	1.220585
direct1	2.288295	.1853601	10.22	0.000	1.952368	2.682022
indirect2	1.153977	.0309939	5.33	0.000	1.094802	1.216351
direct2	2.294933	.1868413	10.20	0.000	1.956454	2.69197

Does it matter?

Table: Comparing different estimates of the size of indirect effect relative to the size of the total effect

	generalization	(Erikson et al. 2005)	naive
middle v. low			
method1	.1359	.1107	
method2	.1343	.1088	
average	.1351	.1098	.0087
high v. low			
method1	.1459	.1107	
method2	.1427	.0990	
average	.1443	.1048	.0142
high v. middle			
method1	.1500	.1075	
method2	.1470	.0968	
average	.1485	.1021	.0167

Discussion

This is “an area of active research”

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There are unanswered questions:

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- The need to take the average indirect effect is less than elegant.

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- The need to take the average indirect effect is less than elegant.
- How does it relate to the alternative method proposed by Fairlie (2005) and implemented by Ben Jann as the `fairlie` package?

References

Buis, M. L.
Direct and indirect effects in a logit model.
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