Analyzing Inequality of Educational Opportunities using Stacked Surveys with Missing Data

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Outline

- baseline model
- Missing Data
  - Multiple Imputation of multiple surveys
  - assess plausibility of results
- Nesting within surveys
  - Random effects model
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- Individuals are nested in surveys (50 surveys)
  - Potential bias
  - Too efficient

Conclusions

- Missing data
  - Virtually no bias was found.
  - Virtually no gain in power was achieved by using Multiple Imputation.
- Nested structure of the data
  - Outlying studies have lead to an underestimation of the trend in IEO in pooled regression.
  - Standard errors increases a little when controlling for nested structure.

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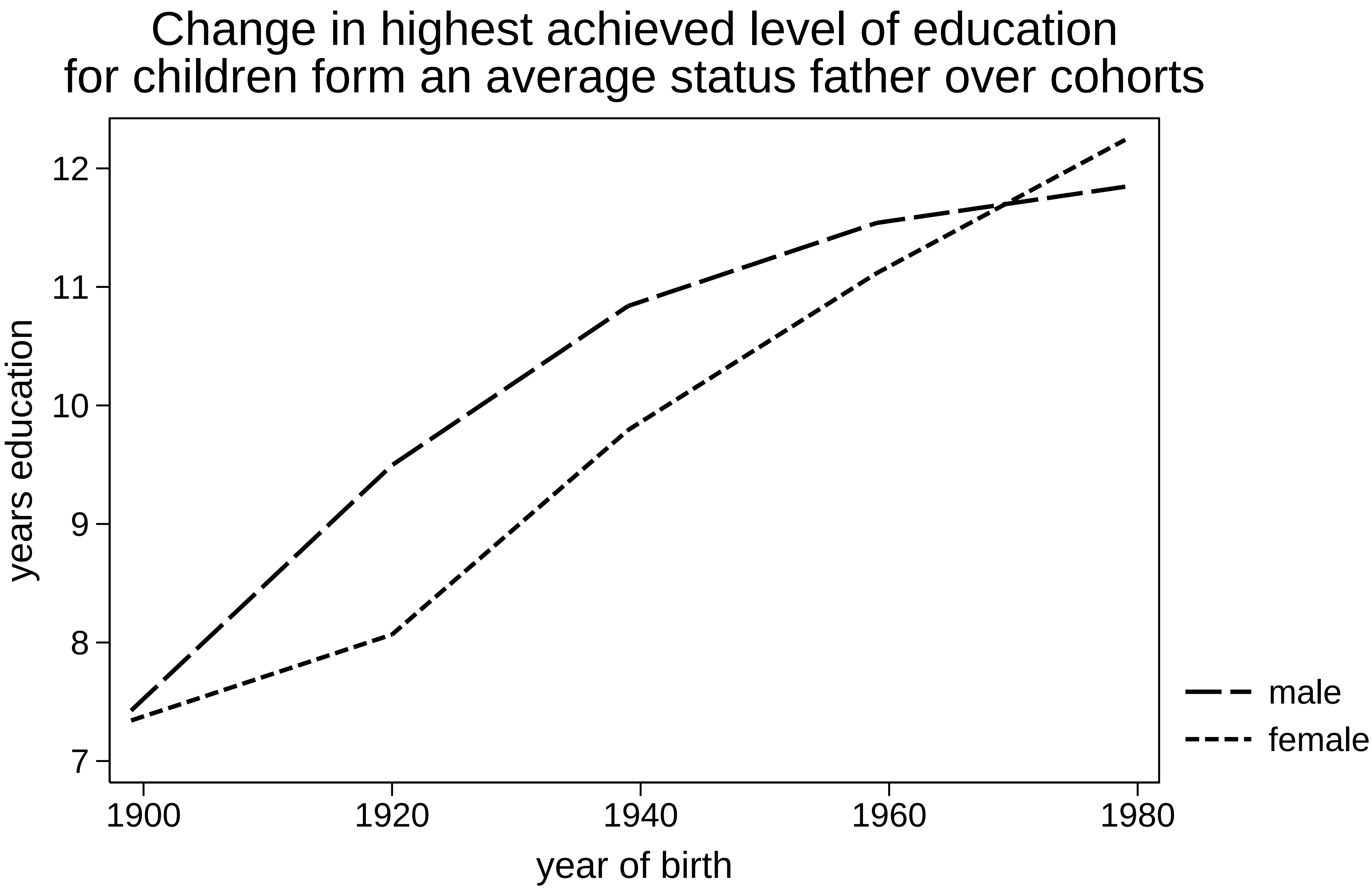
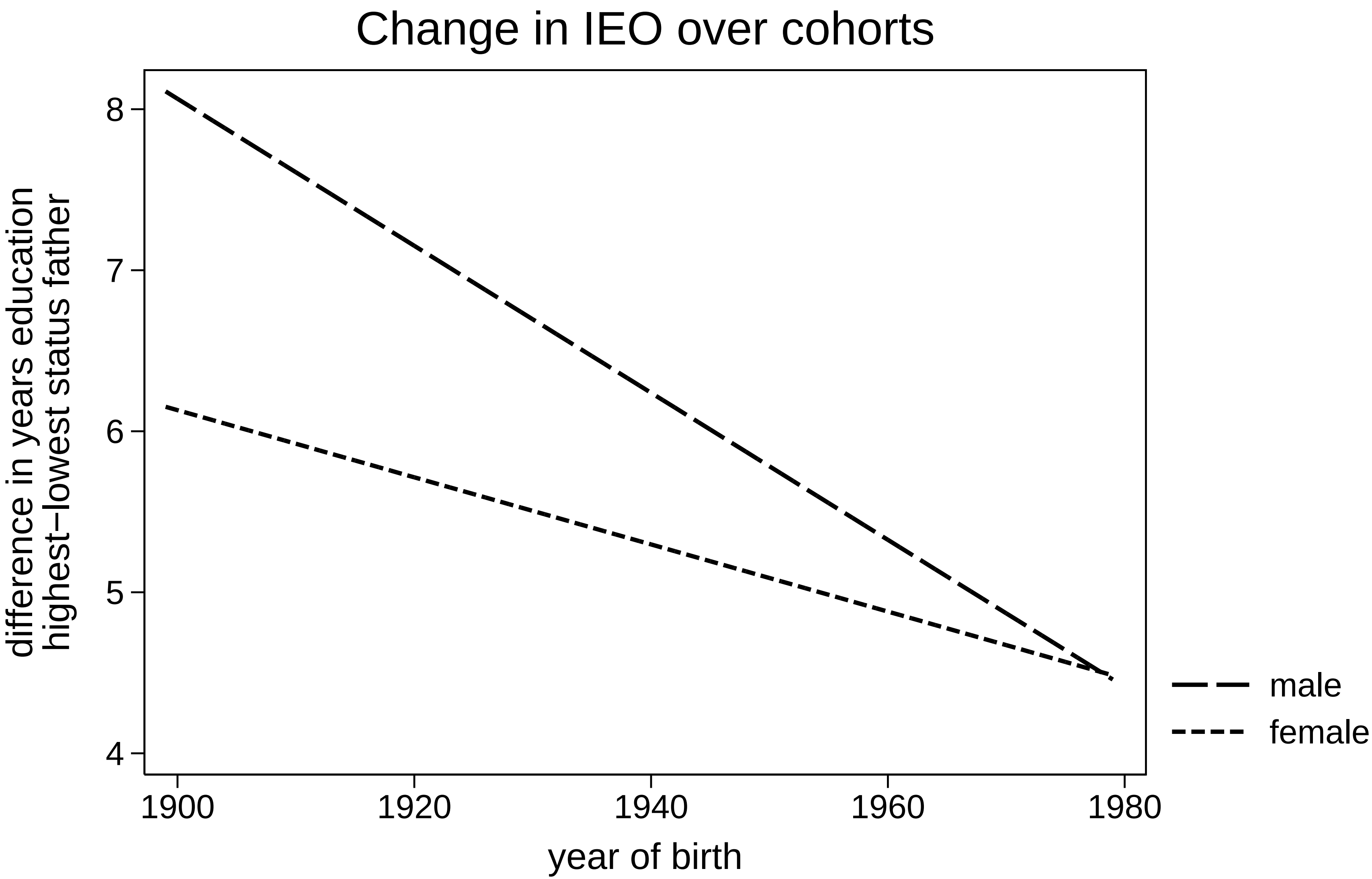
  - an interaction between $status$ and $birthyear$, which captures a linear trend in IEO,
  - and interactions of all variables with $female$.

Educational expansion in baseline model



IEO in baseline model



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- The correction is based on the between dataset variance of the point estimates.

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- Separate models are estimated for each combination of survey, gender, and three year birthcohort to include all interactions and control for differences between surveys.
- Imputations are only made if enough complete observations are available (number of variables + 2).
  - Of 10,617 missing cases for $status$ 10,340 could be imputed.
  - Of 1,145 missing cases for $educyr$ 968 could be imputed.

Multiple Imputation results

		Complete Cases		Multiple Imputation	
		b	se	b	se
Male					
	$status$	8.065	0.252	8.038	0.252
	$birthyear X status$	-4.565	0.498	-4.554	0.500
Female					
	$status$	6.131	0.255	6.165	0.256
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Diagnosing Imputation model

Asses the plausibility of results:

- How plausible is it that some standard errors in imputed model are larger than the standard errors in the complete case model?
- How plausible is it that the parameter estimates in the complete case model aren't biased?

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  - the variance of the explanatory variable itself.
- Changes in these estimates may cause the standard error to go either up or down.

Decomposition of change in SE

Decomposition of change in SE relative to Complete Case SE

	sample size	imputation uncertainty	change in estimates†	total change
male				
$status$	-4.74%	0.25%	4.48%	-0.01%
$birthyear X status$	-4.74%	1.58%	3.46%	0.30%
female				
$status$	-4.74%	1.84%	3.35%	0.45%
$birthyear X status$	-4.74%	1.35%	2.58%	-0.81%

† standard deviation of the errors, degree of multicollinearity, and the variance of the explanatory variable

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- $\text{Pr}(M_x = 0)$ can be estimated by the proportion of complete observations.
- $\text{Pr}(M_x = 0|y)$ can be estimated using a logistic regression of M_x on y.

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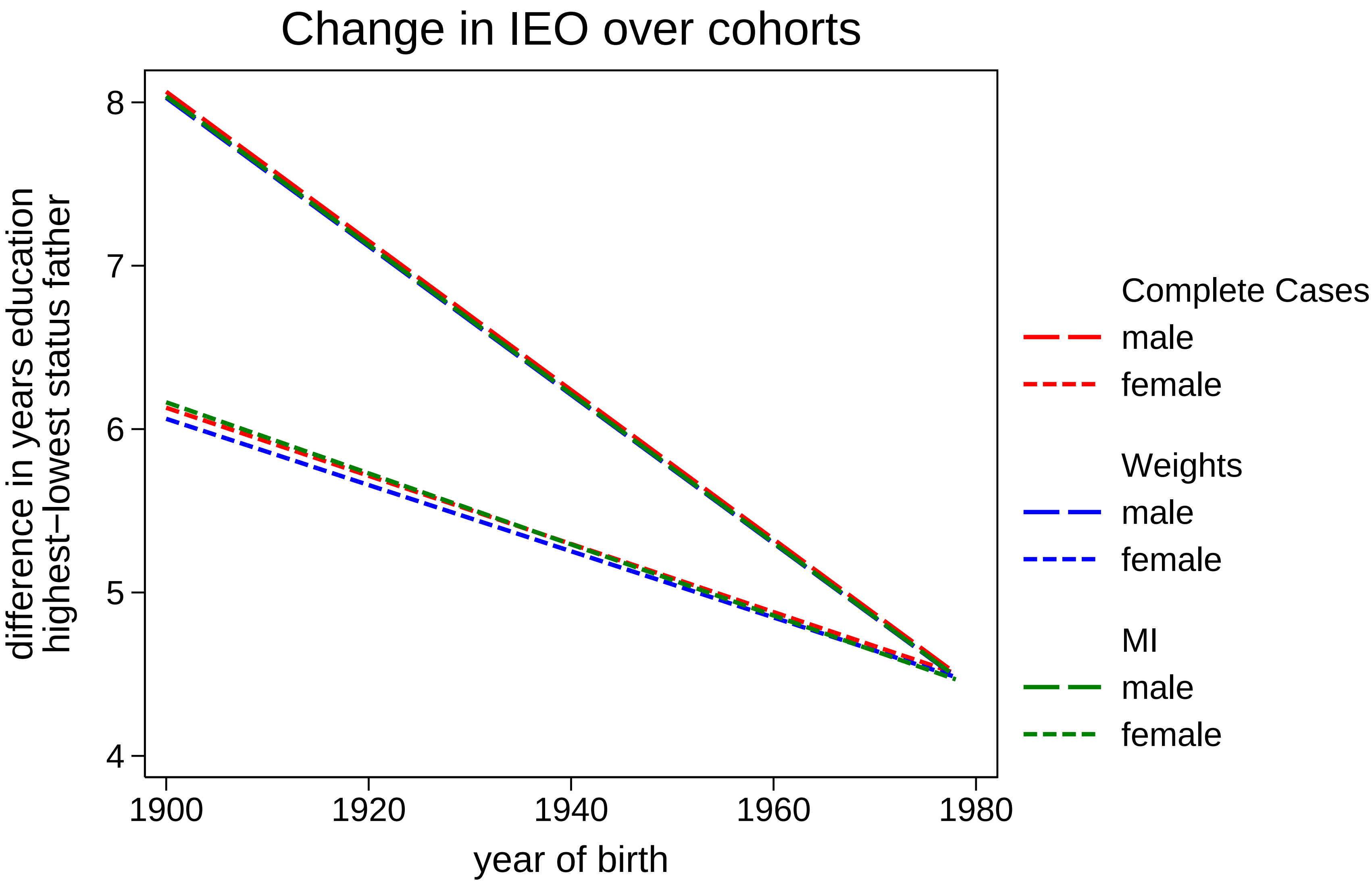
Weighting to correct for bias

This approach can be extended to include:

- missing cases in y,
- multiple xs with or without missing cases,
- interaction terms.

IEO with corrections for missing data



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Nested structure of the data

Random effects model:

- Random effects:
  - (Male) constant
  - (Male) $status$
- Fixed effects:
  - $female$
  - $femaleXstatus$
  - splines of $birthyear$
  - trends in $status$

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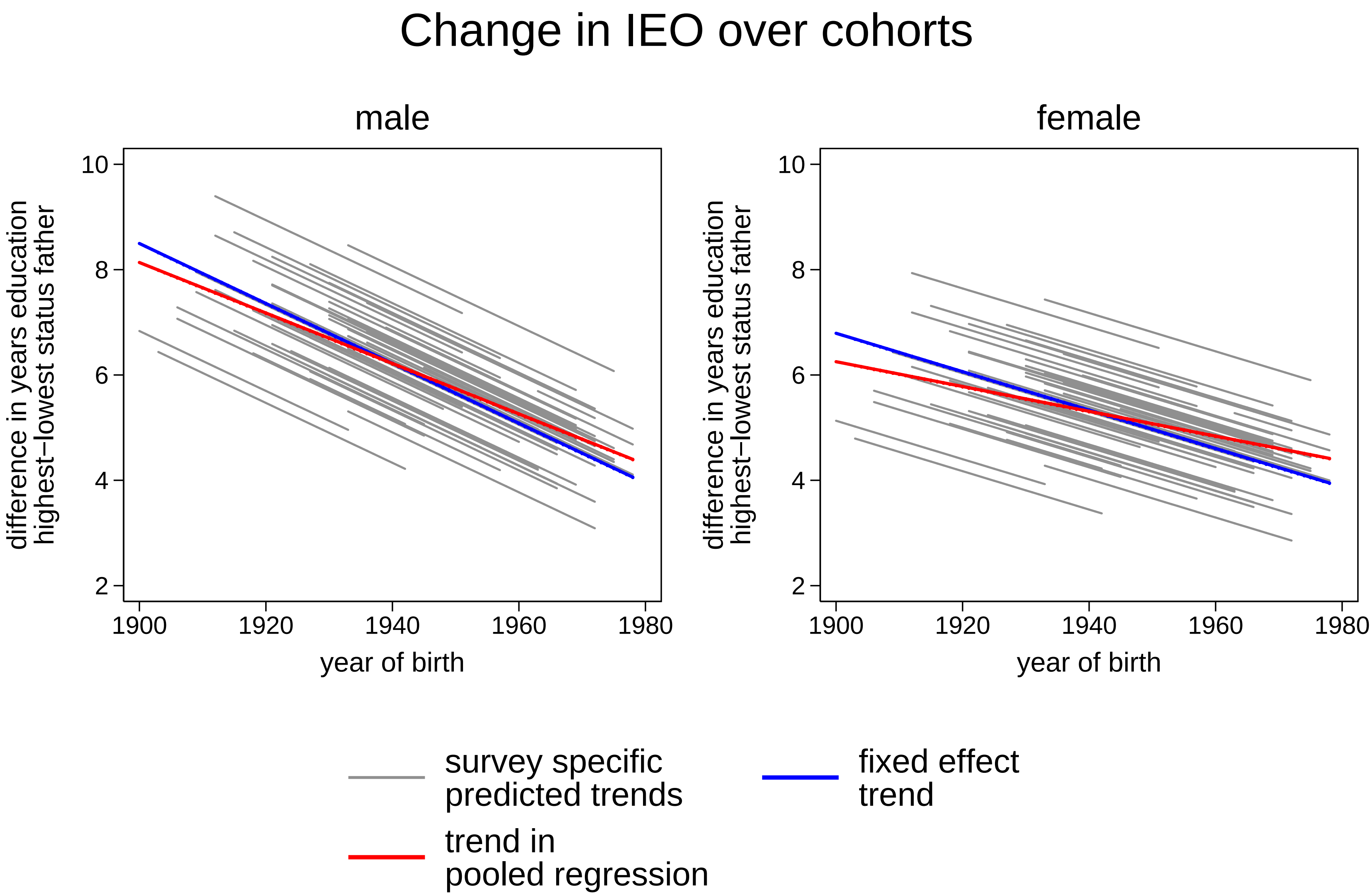
Plausibility of bias in Pooled regression



Outlying surveys

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- Trend in inequality within surveys is pretty consistent.
- These studies provide valuable information about the trend once one controls for level of IEO.

Conclusions

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