Relationship between the likelihood functions used in `fmlogit` and `mlogit`

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`fmlogit` uses the same likelihood function as the multinomial logit model. We need a special program to estimate this model and can't use [R] **mlogit** because of the way the likelihood function is implemented in `mlogit`. We usually think of `mlogit` as estimating as estimating the effects on a single dependent variable consisting of multiple categories, but we could also represent this dependent variable with a set of dummy variables, $y_0 \cdots y_J$. These variables contain only the value 0 or 1 in `mlogit`, while it contains the proportions in fmlogit.

Both models assume that the predicted values (probabilities in `mlogit` and proportions in `fmlogit`) depend on the explanatory variables x through equation (1).

$$\begin{aligned}
\widehat{y}_{0i} &= \frac{1}{1+\sum_{k=1}^{J}\exp(x_i\beta_k)} \\
\widehat{y}_{1i} &= \frac{\exp(x_i\beta_1)}{1+\sum_{k=1}^{J}\exp(x_i\beta_k)} \\
&\vdots \\
\widehat{y}_{Ji} &= \frac{\exp(x_i\beta_J)}{1+\sum_{k=1}^{J}\exp(x_i\beta_k)}
\end{aligned} \tag{1}$$

The log likelihood function is a function of the predicted values:

$$\ln(L_i) = y_{0i}\ln(\widehat{y}_{0i}) + y_{1i}\ln(\widehat{y}_{1i}) + \cdots + y_{Ji}\ln(\widehat{y}_{Ji}) \tag{2}$$

Notice that in a multinomial logit model the ys are just 0 or 1, so in that case their function is to pick, for each individual i, which $\widehat{y}$ should enter in the log likelihood function. So for the multinomial logit we could rewrite equation (2) as:

$$\ln(L_i) = \begin{cases} \ln(\widehat{y}_{0i}) & \text{if } y_{0i} == 1 \\ \ln(\widehat{y}_{1i}) & \text{if } y_{1i} == 1 \\ \vdots & \\ \ln(\widehat{y}_{Ji}) & \text{if } y_{Ji} == 1 \end{cases} \tag{3}$$

This is how the likelihood is implemented in `mlogit`, which is why we cannot use `mlogit` to estimate a fractional multinomial logit model, but if we maximized equation (2) there would be no problem. This is what `fmlogit` does.